written by John Rennie
when a Student at Edinburgh 1782

Pittenweem water Gin -- 1777

From this way of raising Coals says Mr. Hall it requires 16 Tuns 10 Cwt. of water to draw 8 Tuns 2 Cwt. of Coals --

Then says he to draw 100 Tuns of Coals ⅌ day for 5 days of the week will require each day 204 Tuns or 816 Hogsheads

I find says Mr. Hall that a 6 Inch bore of a pump with a 6 feet Stroke 10 Strokes ⅌ Minute will raise 67 Hogsheads ⅌ hour / by Theory 69.06 hence the water that escapes by the Side of the Boxes & Valves is = 2.06 Hogsheads ⅌ hour / and in twelve hours 804 Hogsheads hence it appears that the above Six Inch bore going for little more than twelve hours ⅌ day will draw as much water into the Cistern as will draw the full Quantity of Coals viz 100 Tuns ⅌ day

The additional expence to the Engine keeper is the Leathering of the boxes of a 6 Inch bore of a pump that goes 5 days ⅌ week but to allow every thing full we will suppose the Engine to go 7 days or the whole week -

Mr. Halls Fire Engine has at present he says Sufficient power to work a 6 Inch bore of a pump over & above her other pumps & therefore the drawing of the Coal will require the Engine to go no longer than usual -

 as the quantity of Coals that is requir'd is 500 Tuns
ℙ week the Matter then is reduced to this Simple question
viz Supposing a pump of Six Inches bore to go 12
hours every day for 5 or to make the full allowance ℙ day
ℙ week how much aright the Engine keeper to
get for his trouble in leathering the Boxes —

A description of the Above mentioned Machine

This machine is constructed like a horse Gin for raising
of Coals with two Seperate Ropes going over pullies or
wheels at their respective pits to the ends of the one Rope
is fastened two water buckets & to that of the other
the Corf in such a manner that when the one bucket
descends full of water the one Corf is ascending full of
Coals & the other water bucket is ascending empty
& the other Corf descending empty at the bottom
of each water bucket is a Valve with an Iron rod
fastened to it that goes below its bottom & consequent
-ly when the bucket arrives at bottom its weight
resting upon the Iron Rod presses it upwards
opens the Valve & So emptys the bucket of its water
the Corf full of Coals is now taken off & an empty one
put on & at bottom the empty Corf is taken off

& a full one put on upon which a Signal is given & the man at top fills his bucket with water which immediatlly descends by its gravity raises the full Corf & the empty water bucket ~~which~~ ~~the~~ & carrys down the empty Corf & so on alternatly ascending & descending. as the Rope that is fastened to the buckets is of a Considerable weight & as the bucket that descends carries the Rope along with it whilst the other one is Shortning its weight would Accelerate the velocity of it so much as to endanger the bucket or perhaps the workmen employed to keep therefore the motion equable there is fastened to the ~~bottom~~ of each water bucket a Chain whose weight is equal to that of the Rope descending & so keeps the motion of them equable.--

as it is necessary the Gin should be ~~eased~~ a little before the Corf at top can be unhooked there is four Levers in Axle of Gin so long that a man is able to raise or ease the Corf with his hand untill it is taken of.. And in Case of any accidents such as the breaking of a Rope &c. there is a Sed Wheel or break by which ~~it~~ may be wholly Stoped.

Experiments made on the City of Edinr. pipe Conducting the water from to

yards from the source	depression in feet	water ½ minute pints Gills
200	29.6	42 . 3
423	55	48 . 3 - 23
581	55.	40. -
1000	30.	21 . 7
1200	35.	25 . 10
1320	75.6	26 . 14
1432	100.	28. -
2143	133	27. 12

These experiments says Prof. Robison was made on the old pipe at Swanstone the pipe was cut over at the different Lengths & run with a full mouth.
N. B Mr. Selby says its diar. was 2½ Inches

Dimensions of some Merch.ᵗˢ Ships as taken at Dunbar 18.ᵗʰ Sept.ʳ 1783.

	Length over all	Breadth over all	Draught of Water when loaded
a Brig of 280 Tons	78 f.	22 f.	14½ f.
a Brig of 160 d.°	66	19	from 10 to 12
a Sloop of 90 d.°	60	17	10½
a Sloop of 58 d.°	46	15	7½

An Account of the principal Machines used for blowing Air into Furnaces by a fall of Water with experiments & Observations on the same as describ'd by Mr. Lewis in his

Commercium Philosophico-Technicum

Sect. I.

1 A Simple pipe.

The first account I have met with of a machine for propelling air into furnaces, by a fall of water carrying down air with it, is of one at the Copper and brass furnaces at Tivoli near Rome, of which a description & figure are given in the 3d number of the Philosophical Transactions, & in the Journal des Savans for the Year 1666.

A square wooden pipe, of considerable width, & open at both ends, is placed upright. A Stream of water runs in at the top, & is discharged at the bottom; and about the middle of the height of the pipe a smaller horizontal one is inserted, which reaches to the furnace, & is said to convey to it a strong blast of air.

From so imperfect a description, we can learn little of the nature of the machine, or of the manner in which the blast is produced. It may be presumed that the water, running forcibly against the side of the pipe, as it ap-
-pears.

appears to do in the figure, is in great part dashed into drops: the intervals between which being filled by air, this air is successively pushed down by the drops that follow, & afterwards escapes as soon as it meets with a vent. There seems, however, to be either some inaccuracy in the description, or some essential part omitted: for in such trials as I have made, when air, thus conveyed into ~~a pipe~~ a perpendicular pipe along with running water, was discharged by a lateral ap aperture, part of the water always accompanied it in a stream; & more of the water seemed to spout out in proportion as the quantity of air introduced was the greater.

II A pipe with air holes, inserted into an air vessel

M. Belidor in his Architecture hydraulique, gives a more particular description of a water machine used in some parts of France; he says there are four or five forges on the River Isere, between Romans & Grenoble, which have no other bellows.

The stream is divided into two channels, & each division falls into an upright pipe ten or twelve feet high. Near the tops of the pipes are several holes, made sloping downwards from the outside to the inside; thro' these holes air enters. & is carried down by the water; though the experiments in the following section will show, that the quantity of air thus introduced is not so great as in the dispositions mentioned hereafter.

The essential difference of this instrument from the foregoing consists in its having an air vessel, or reservoir for the air, at the bottom. An oval wooden tub, near 7 feet high, & 3 or 4 feet wide, is inverted, & its lower edge let into the ground 5 or 6 Inches. The lower ends of the two upright

pipes enter into the top of the tub. & under each pipe is a kind of small Stool which the water falls on. The water loaded with air, dashing against the Stool with great Velocity, re-bounds, & its air is discharged or disengaged: a pipe communica-ting with the top of the tub carries the air to the furnace. While the water runs out at a hole in the lower part; a suf-ficient height of water being keept in the tub, above this hole, to prevent any air from escaping by it.

III A funnel & pipe without Air holes, inserted into an air Vessel.

M. Mariotte, in his treatise du mouvement des eaux, gives an account of another contrivance for blowing fire by a fall of water, which Belidor says, from the information of a friend who travelled in Italy, is used in the Tiburtine mountain near Rome, & near Salan on the lac de Guarde. A wooden or tin pipe, 14 or 15 feet high, & one foot diam.r has its lower end fixed into an air Vessel or inverted tub, as in the preceding article, from one side of which a blast pipe goes tapering into the furnace. –

The upper end of the large upright pipe is contracted to an aperture of 3 or 4 Inches, into which is fitted a funnel, whose neck exactly fills it. Into the funnel there falls a stream of water, from the height of 10, 15, or 20 feet: which we may pre-sume to be dashed into drops in its fall, & push down air before it on the same principle as in the machine of Ti-voli already mentioned.

This instrument promises to be more effectual than either of the preceding, though in this country it can be of little use so high a fall of water being rarely to be procured, at least in those places where smelting furnaces are established. – –

IV A funnel & pipe with air holes, inserted into an air Vessel.
At Lead hills in Scotland. –

In No 576 of the Philosophical Transactions, in the year 1745, Mr Stirling describes a Machine erected in Scotland, for blowing air into the furnaces in which Lead ores are Smelted; & for conveying fresh air into the works, so as to save the trouble & expence of the double drifts & Shafts, & the cutting of communications between them.

A Stream of water runs into a wooden funnel, so as to keep it always nearly full; the height of the funnel is 5 feet, & the diar of its throat 3 ½ Inches. The neck of the funnel is inserted into an upright pipe, whose diar is 5 ½ Inches & its length 14, 15, or 16 feet. immediatly under the throat of the funnel, four air holes are made in the pipe, at equal distances round it, about an Inch & a half wide. Sloping downwards from the Outside to the inside.

The lower end of the pipe enters into a wooden tub, close at top, but without a bottom, Six feet high & five & a half wide, Sunk into pit dug in the ground, & well rammed about with clay: in the middle of the tub, directly under the pipe, is a flat Stone about two feet high, for the water to fall upon; & into the top of the tub is fixed a wooden pipe for carrying off the air, communicating at the further end with an Iron one which enters the furnace; for regulating the blast, a small hole is made in some convenient part of the pipe, which is stoped with a pin, or opened, according as the blast is required more or less strong. The hole in the lower part of the tub, by which the waste water passes out, is about 5 Inches Square; & one side of the pit, where the water runs off,

is a little lower than the Surface of the Stone, so that the water can never rise high enough to cover the Stone; though it is supposed to continue always a considerable height above the top of the hole:

Though this Machine is said in the Transactions to be sufficient for the Smelting of harder ore than any in Leadhills where it was erected, I have been informed by a person concerned in these works, that it has since been found not to answer so well as could be wished, & that accordingly it has been laid aside, & its place supplied by the common bellows. -

In Dauphiny in France

The blowing Machines used in Dauphiny for the forges & smelting furnaces have a great resemblance in their general Structure to the foregoing. they are described by Swedenbourg in the 2d Volume of his Regnum Subterraneum, but with little exactness: a more accurate description & figures of them, taken from the papers left by Reaumur, are inserted in the Art des forges & fourneaux à fer, published last year by the direction of the French academy.

The upright pipe is generally between 25 & 26 feet high: it is composed of two pieces of fir, hollowed, & joined together by Iron work. Instead of a distinct funnel or reservoir on the top, a part of the pipe itself is hollowed so as to perform the same office: at the top it is 12 Inches & 1/5 diam. (English Measure) & from thence it grows narrower to the depth of nearly 34 Inches, where its width is only about 3 3/4 Inches: immediately below this part called the choak, its cavity widens to nearly 8 1/2 Inches, & this width it preserves throughout the rest of its length. Under the choak are 10 air holes, six of which are in one horizontal plane, at equal dis-

-tances

from one another & the rest about 3 1/4 Inches lower down: all the holes are Cylindrical, near two Inches in diar., & cut at such an Obliquity, tht the orifice of the upper ones is on the inside of the pipe Eight Inches, & on the outside only 5 Inches, below the choak.
The tub or air Vessel, which receives the lower end of the pipe is 5 1/2 feet, ~~high~~, or a little more, in depth, & nearly as much in width: the pipe enters into it about seventeen Inches: about the middle of its height is a flat Stone or Iron plate, supported by cross barrs of wood. The air passes off, as already mentioned, through a pipe inserted into the upper part of the tub, & the water thro' a hole at the bottom: on the outside of this hole is fixed a wooden frame, with an upright Slider, by which the aperture for letting out the water may be occassionally increased or diminished. The blast is regulated, & the air suffered to escape when it is not wanted, by a hole in the blowing pipe, to which is fitted a Valve or a Stopper.
One of these machines is said to be sufficient for the forge or Iron finery, & two or three for the furnace in which the Iron Ore is run down...

In Foix in France.

In the County of Foix, the blowing machines, as described by Reaumur in the art des forges above mentioned or quoted, are considerably different from the foregoing. The pipe is rectangular, & the part above the choak divides into three funnel shaped partitions. On the top is a reservoir or Cistern of water; & two of the partitions, close on all sides, pass up above the surface of the water, for carrying down air, & thus supplying the place of lateral Air holes; the

water enters into the third partition, which is only the space between the two foregoing, & which has but two sides, formed by the two opposite sides of the others.
The Author makes the principal difference of these Machines from those of Dauphiny to consist in this disposition of the upper part: but the plate, annexed to this description shows another, which is, perhaps, more material to the effect of the instrument. The whole height of the pipe, including that of the water in the reservoir on the top is, according to the scale, 20 or 21 feet, & the choak or narrow throat is almost down at the middle of this height: so that the water issues thro' the choak with a velocity which it acquires from a pressure of about 10 feet, which is greater than in the Machine of Dauphiny in the proportion of about 11:6: the quantity of water seems also to be much less in proportion to the width of the pipe, the great pressure probably occasioned it to spread, so as to fill a larger bore than it could do when falling with less velocity. Two pipes, divided in the same manner at the top, are fed by one reservoir: the lower ends of the pipes enter into one large oblong box, from which the air & water pass out as in the forgoing Machines -

A St. Pierre in Languedoc.

Mr. Barthes, in a curious paper printed in the third volume of the memoirs of the correspondents of the French Academy, gives a minute description, tho' in some parts not so clear as could be wished, of a blowing machine at the forge of St. Pierre on the river Obriou, which he looks upon as one of the most perfect of

the

the instruments of this kind. Its general Structure is nearly the same with that of Foix, but the height of water above the choak much less. The upright pipe is square, about 9 ft high, & somewhat more than Seven Inches wide. Into its top are inserted, at opposite sides, two pyramidal Air pipes, widening upwards, and passing up Obliquely through a bason of water four feet high. The Space included between the pipes, at the lower end, under the bason, is a kind of hopper, into which the water enters through two apertures in the bottom of the bason: to each of these apertures is fitted a piston or Stopper, hung to the end of a lever, by which it is raised more or less, according as more or less water is required. Two of these instruments are furnished with water from one bason; & the lower ends of both enter into one Air Vessel, which is near five feet high, about 6 ft long, near 3 ft wide at one end, & not quite two at the other. The Stones, for the water to fall upon, are some-what less than 4 ft inches distant from the pipes: the water runs off through two rectangular apertures at the bottom, each about 8 ft Inches wide, & near 6 Inches high: the pipe which carries of the air, is an Inch & quarter diameter at the small end where it enters the furnace.

The Obscure part of the description relates to the hopper, & the apertures by which the water is discharged from it into the perpendicular pipe. The hopper seems to be divided into two upright partitions; & there are "two horizontal rectangu-lar openings, through which the water runs into the two hop-pers, each of them about 7 ft Inches long, & 5 ft Inches wide, measured on the level of the bottom of the reservoir, which width is reduced to 4 ft at the extremity of the air pipes, where the hopper also terminates.

The Author Observes that in this machine, the water, issuing from the hopper, is necessarily reduced into drops. To Satisfy

himself more fully of this particular, he took a tin Vessel, 8½ Inches Square, & six & one half high: in the middle of the bottom he cut a rectangular opening, about an Inch & a tenth long, & 8/10 wide: to the two long Sides of the Slit he solderd two tin plates, inclined to one another, & a third across them. These apertures, he says, represent those of the Machine when the Stoppers are drawn up; & water put into this vessel came out always, during the whole time of its running, in Streams which struck against & crossed one another, & which, after spreading, were reduced into drops. ——

In this illustration of the Machine, though it seems clear, there must be something which escapes my apprehension; Having cut an aperture of the above dimensions in the bottom of a Vessel, I fitted to each of the longer sides a plate half the width of the aperture, both of which plates were moveable, and kept at different inclinations by means of the third plate which passed across the middle of the two. The Vessel being filled with water, I could not observe, as indeed was expected, the least crossing of the Streams that run through it: on the contrary greatest part of the water issued in two opposite directions, horizontally, from between the ends of the plates. ——

Sect. II.

Experiments & Observations for the improvement of the foregoing Machines, and for establishing their principles of Action: ..

I Of the quantity of water they require, and the quantity or force of the air they afford. ——

The quantity of water may be estimated with sufficient exactness, from the height of the water in the funnel or bason on the top, & from the width of the choak or throat of the funnel,

through which it is pressed by the force of a column of that height.

Desaguliers found, by an experiment often repeted, that the quantity of water running thro a hole one inch square, 25 inches under the surface, is five tuns & one fifth in an hour, the ton containing 252 gallons. The quantities discharged thro' equal holes at different depths being as the square roots of those depths, & the quantities thro' different holes at equal depths being as the areas of the holes; it will appear on calculation, that in the Machine at Lead Hills, whose funnel is five feet high, & its throat 3½ Inches in diameter, the expence of water is somewhat more than 77 tons in an hour, or near 324 Gallons in a Minute; & that in the Machine of Dauphiny where the height of water in the funnel is only about half as great, and the bore of the throat a little wider, the quantity of water is about 266 gallons in a Minute. Perhaps the real quantity of water may be somewhat less than this calculation gives, as the resistance of the compressed air may occasion some retardation of the motion. Of the other Machines, the descriptions are too imperfect, or Obscure for any computation to be made from them.

The water, issuing from the narrow throat of the funnel with great Velocity, is said to spread so as to fill the wider bore of the ~~neck~~ pipe, & to become frothy from the mixture of air with it. The jet thus enlarged may be conceived as consisting of a Multitude of slender streams or drops, the intervals between them being occupied by air, which is continually supplied thro' the air holes, & pushed down by the

succeeding drops or streams. It has therefore been reckoned, that the volume of air which passes down the pipe must be as much greater than that of the water, as the transverse area of the jet, when spread & reduced to drops in the pipe, is greater than when it passed through the throat of the funnel. Circles being to one another as the squares of their diam.rs, the area of the pipe of the Leadhills machine will be to that of the funnels ~~throat~~ as 18 : 12·25; the volume of air, according to the above principle, being to that of the water in the same proportion, & the quantity of water nearly 324 Gallons in a minute, the quantity of air in a minute should be about 475·5 Gallons, or 134,000 cubic Inches, or 77·5 cubic feet. In the same manner, the Machine of Dauphiny will be found to yeild about 1080 Gallons, or upwards of 304,000 cubic inches, or 176 cubic feet, of air in a minute: so that by this way of reckoning, the Dauphiny machine, with near a fourth less of water than that of Lead-hills, should produce more than a double quantity of Air. — But tho' this method of computation appears specious, it is not perhaps to be much depended on; air, in different circumstances, occupying very different volumes, in virtue of its great compressibility: nor is it certain that the bores of the pipes are sufficiently filled, so as to carry down the full quantity of air. It may be presumed, that the air, intermingled in the jet, is always in some degree compressed by the water: so that the interstices between the streams or drops contain more air than equal spaces of the atmosphere. It may be judged however from the above comparison, that the wider the pipe is,

in proportion to the funnel's throat, provided the water running through the throat will spread through the whole extent of the bore of the pipe, the more air will be carried down.

Mr. Barthès, the only person I know of who has examined these Machines Philosophically & endeavoured to improve them, gives a method, in the memoir above quoted of comparing the proportional quantities or forces of the air in different blowing Machines, on another principle. From considerations too abstracted to be here particularised, he deduces a general rule, that the produce of air will be in all cases in proportion to the quantity & velocity of the water: So that the quantity of water & height of the fall being given in two Machines, & the volume or force of the air afforded by one of them being measured by experiment, the volume or force of the air in the other may be determined by the rules. Accordingly he made several experiments of this kind in two Machines; measuring the force of the air, when the water in the bason was at different heights, by the weight, which the blast acting on the arm of a balance, was capable of raising. Taking one of these experiments for a standard, he computed by the rule what the results of the others ought to have been; but the experiments & calculations agreed ill together. And indeed the rule does not seem to be applicable but in circumstances, which can scarsely be expected to occur; for it supposes the Machine to be all perfect, & every drop of the water to have its ut-

most

-most effect, or to carry down with it as much air as it is capable of doing; which cannot be admitted to be the case in any of the blowing machines yet constructed — —

In the art des forges are mentioned some observations of Reaumur of the quantity of air afforded by the wooden bellows. He finds that those used at the Iron furnaces yeild 98280 cubic inches, or upwards of 5 cubic feet of air at every stroke; and including the two bellows which act alternatly, 240 strokes in a quarter of an hour; which on a reduction of the French measures to the English, make 130896 cubic Inches, or upwards of 75·3 cubic, feet in a minute: this quantity exceeds that which the forgoing calculations gives for the machines of Dauphiny above four times, & therefore four of the machines should scarcely be able to supply the Iron furnace with so much air as the wooden bellows does; whereas two or three are said to be sufficient. Again, the bellows of the Iron finery A forge was found to give 2031·33 cubic inches at one Stroke, & 412 Strokes in a quarter of an hour; whence the quantity of air in a minute is 458,247 cubic inches or somewhat more than 265 cubic feet: this is greater than the calculation of the water machine, in the proportion of about 3:2, tho' one of the water machines is found to supply the office of the bellows.

It is not to be supposed, that the quantity of air, which furnaces require, is confined to any such precise limits, as that two bellows, from their being found to answer sufficiently for one kind of furnace, or even for one individual furnace, can be concluded to yeild quantities of air exactly or nearly equal. The

The above differences are perhaps as little as can be expected in comparisons of this kind where the effects compared are so indeterminate. ——

As to the Water Machines, it is plain, that the quantity of air carried down cannot be greater, than the Spaces between the drops or divided Streams can contain; & that though the air in these Spaces must be considered as being compressed to a certain degree, yet it cannot be supposed compressed into two thirds its natural Volume, which would be necessary for making the calculations of the wooden bellows & blowing machines to agree, because such a condensation would require the weight of a column of water 11 or 12 f.t, or the third part of such a column as is equivalent to the pressure of the Atmosphere; whereas in the Dauphiny machine, though the air was pressed down with the full force of the column of water above the choak, the height of this column is less than three feet, & could not condense it more than one twelfth part - - -

In what manner Reaumur computed the air of the wooden bellows, we have no account, it is probable that he judged, as others have done in the same cases, from their capacity; supposing the whole quantity of air they contained to be delivered at every Stroke. If so, we can lay no stress on the computation, for neither the wooden or the leather bellows deliver their full contents of air; a considerable Space remaining full of air when the bellows are closed; and this space containing considerably more air than an equal volume of the atmosphere, on account of the air being condensed in it by the pressure of the bellows. I have been informed by a judicious workman, that the bellows of the Iron finery retains commonly a third, & sometimes half of its air; and that when lined with wood, so that as little vacant space as possible might be left, he found it to blow much stronger than before. — —

The Strength of bellows is best judged from the force of the blast itself; & this force may be determined, in the method recommended by M.r Barthes, already mentioned, by the weights it is capable of raising. He found that in the blowing machine of

St. Pierre, described at the end of the preceding Section, the force of the blast issuing from a hole one Inch & one third diar. raised the arm of a balance loaded with a weight of 25½ ounces. He gives some other experiments, of comparing the proportional diminution of its force according to the dimi-nution of the height of the water; which I shall here insert in the original french measures, to avoid unnecessary fractions. The above force of 25½ ounces is the maximum of this machine, produced by the full quantity of water in the bason, or a height of 48 inches above the choak: with a height of 41 Inches, the weight raised was 22 ounces; with a height of 32 Inches, 19 ounces; with a height of 28 inches & a half 17·25 ounces; with 24·5 Inches, 15·25 ounces; with 19 inches, 12⅜ ounces; with 16 inches & two thirds, 10·25 ounces; & with a height of 13·5 inches, 8·75 ounces. - - -
It may be observed, that in some of these experiments the water must have been employed to disadvantage; & that by increasing the height of the water much further than the above limits, in the same machine, we could not expect to produce proportional augmentations of the force of the blast: for if a certain quantity of water, running with a certain velocity through the choak, be supposed to fill the bore of the pipe; a less quantity, with a less velocity, must leave a vacancy, which will suffer part of the air to escape; & a greater quantity, with a greater velocity, must have some part of it spent ineffectually, for want of sufficient room to spread. Some experiments mentioned hereafter afford a clear proof of this. - - - -
The force of the air may be determined in an easier & more simple method, by means of a glass pipe, open at both ends, with one end fixed in a bason of water. The bason may be hung in the upper part of the tub or air vessel of our water machine,

and the glass pipe let into it through a hole in the top, what space may remain between the pipe and the hole being properly closed: the pressure of the air on the surface of the fluid in the bason, forces part of it up into the pipe; and this ascent will always be the measure of the power or density of the air. Water here is greatly preferable to the quicksilver used in the same intention on other ~~experiments~~ occasions, as it discovers smaller variations in the force; for being 14 times less ponderous than quicksilver, an equal pressure forces it 14 times higher in the pipe: the whole ascent of quicksilver, by the pressure of the air in the bellows, is so small, as frequently not to exceed that part of the ~~tube~~ pipe which is inserted into the tub. Instead of a glass pipe, a copper or iron one may be used; & the ascent of the water measured, either by occasionally dipping a rod in it, or by means of a hollow copper ball, or other floating body, with a stem standing out of the pipe, and a proper weight below to keep it upright. It must be observed, that the height of the water in the pipe is to be estimated from the surface of the water in the bason; whence the pipe ought to be of small bore in proportion to the bason, that the water may not fall considerably in the bason by the loss of that which rises in the pipe. – –

Dr. Hales found that a smith's bellows raised a mercurial gage about an inch, so that it would have raised a water gage about 14 Inches. The twenty five ounces & a half, raised in Mr. Barthes's experiment by the blast of the machine of St. Pierre from an aperture from an aperture of an inch & a quarter bore, English measure, are equivalent to the ascent of water in the gage pipe 40 or 41 inches. I have been informed, that the pipe by which the air is discharged into our iron furnaces is at least of an Inch & a half bore; and that the air, with this aperture to pass off by, ought to be of as great density as it can be reduced to by the human breath in a confined space; which is such as to raise the water in the gage about 50 Inches;

in which case it is compressed into near an eight part less volume than it commonly occupies in the Atmosphere. But the quality of the fuel & other circumstances occasion such variations in this respect, that no general standard can be laid down. I have been assured, that a charcoal fire will be excited as strongly by such a blast as raises the gage 36 inches, as a fire of coaked pitcoal will by by one of fifty inches. - - - -

II Observations on the Air Vessel.

The structure of the air vessel, or tub at the bottom, is in great measure independent of that of the rest of the instrument; the same air vessel serving equally for different kinds & sizes of these machines, while the perfection of the other parts consists in their adjustment & proportion to one another. The office of this vessel being only to serve as a reservoir for the air, & to suffer the waste water to pass off, no great care seems to be needfull for regulating its dimensions; & as the stone, which is placed in it under the pipe, serves only to receive & support the fall of water, or to occasion the water to be dashed into small particles, that the air may be more effectually extricated, its distance from the pipe seems also to require no exact adjustment. There are however some particulars, in regard to the size of this vessel, & the disposition of some of its partes, which appear to deserve attension. The gage, mentioned in the preceding article, will be an useful addition to it; shewing at all times by inspection the force of the blast, & thus enabling the workman to judge whether it is sufficient for the purposes intended, & giving him notice of any failings or imperfections that may have happened in the machine; as whether any air escapes through the joints

or cracks, or whether the choak or throat of the funnel is obstructed by stones or other matters brought by the stream.
All the writers I have met with, who give any account of these kinds of blowing machines, seem to suppose the water within & without the air vessel to be upon a level. But as the air in the air vessel is so far compressed, as to be able to raise the water in the gages to a considerable height, it must necessarily act with equal power on the water below it; and if this water can pass off freely at the bottom, it must be depressed as much as that in the gage pipe is raised. The water within & without the vessel is exactly in the same situation with that in the bason & pipe of the gage; excepting only that the former receives a continual supply within, which passes of as fast on the outside. The excess of the height of the water on the outside of the vessel, above that of the water within, appears to be the very power by which the air is compressed & driven into the furnace.
To be further satisfied of this depression of the water, I used, for the air vessel of a smaller machine, a tall glass, without a bottom, 7 or 8 inches of its lower part being immersed into a tub full of water. As soon as the machine began to play & the gage to rise, the water within the glass sunk lower than that in the tub on the outside; & the depression of the water & rising of the gage were, as nearly as could be judged, equal, & kept pace with one another. In a little time the water was forced quite out of the glass, & the air following it rose in bubbles to the top of the tub.
The bottom of the air vessel ought therefore to be sunk at least as much below the level where the external water passes off, as the

gage is expected to rise; for otherwise, before the air is sufficiently compressed to raise the gage to the due height, it will force all the water out below, & in part escape itself by the same aperture. Hence the depth of the air vessel, in any of these Machines where the water has a free passage at the bottom, gives a power which the force of the blast in that Machine can never be made to exceed: thus at Leadhills, the water being only of the height of two feet from the bottom of the Vessel to the level of the bank where it runs off, the air can never be compressed further, than to be able to support a column of two feet water, or to raise the gage to that height; whereas in the Machine of St. Pierre, the compression is about 1/3 greater.—

The sinking of the water in the air vessel may indeed be prevented, by making the aperture at the bottom, thro' which the water is discharged, of such a size, that the pressure of the air may be able to drive ~~no more~~ through it no more water than is received at top. But such an adjustment would be apparently very difficult; & tho' it should be exactly hit, yet, if the quantity of water received was not always the same, it would scarcely be possible to avoid either a depression or elevation of the water in the air Vessel.

Though the depth of the water be sufficient to resist the pressure of the air, it will be easily conceived, that if there was no solid body to support the fall, the great force of the stream, falling from such a height, would push down or dash about great part of the water in the bottom, so that the air would get at the hole, & in part make its escape with the water. It may be presumed that even the drops of water, rebounding from the Stone, & falling down again, have a little effect, tho' in a lower degree; for drops

falling

falling through the common Atmosphere into water, carry air with them, which afterwards rises in bubbles, as may often be observed in heavy rains; & it is not to be supposed, that the drops should not here also carry into the water some of the compressed air, which surrounds them & is entangled between them. Though part of the air, which thus passes into the water, doubtless rises again in bubbles, as appeared in using the glass air vessel above mentioned; yet part may also be pushed so low, as to escape thro the hole, & discover itself by bubbles in the water on the outside of the vessel, which I several times observed before the water was driven interly out of the glass. — —

Mr. Barthes likewise takes notice of air being thus carried down into the water by the drops, or introduced into the cavities which they form in falling. In order to prevent it, he recommends making a partition across the tub, at the level of the stone, with only a hole at one side, & this in the part most remote from the pipe through which the water falls: the rebounding drops are received upon the board, & run off gently through the hole into the water underneath.—

The inconvenience me be prevented also, as effectually, & with more advantage in other respects, by making the air vessel of a very considerable depth below the surface of the stone; it may be sunk several feet into the ground below the level of where the outward water runs off, so as to have always a column of water in the vessel, of any height required, or of a height which shall secure against any air passing down to the bottom. This structure would free the workman from any care about increasing or diminishing the apertures, or regulating the height of the water. For if the deep vessel has an aperture in its lower part, large enough to discharge all the water that can fall into it through

the pipe in the top, or, for the greater security, a good deal larger, its magnitude being of no inconvenience; if this vessel is sunk into a pit of water up to the level of the Stone, or to a certain height a-bove it; & if the pit has a drain sufficient to carry off what more water it may receive: we may be sure that the water will be always high enough in the vessel, because the pressure of the water on the outside will keep it so; & that the pressure of the air within the vessel will always keep it below the Surface of the Stone. — —

The air extricated from the water is always moist: when let off at a little way above the Stone, I have often observed it to leave drops like dew on any solid body opposed to it. A small degree of moisture may perhaps be of no disadvantage; but such a degree as this must ~~un~~doubtless be injurious, & render the air of less efficacy for animating the fire. — — —

In the water machines of Dauphiny, inclined plates are said to be placed at the entrance of the pipe which carries off the air, to keep back the watry drops. Mr. Barthes proposes letting the air off into another vessel, in which spunges are to be hung for imbibing the moisture, & in the bottom of which a cock is to be fixed for occasionally letting off the water that drops from the spunges. I apprehend the intention may be more effectually answered, by making the air vessel of a conside-rable height above the surface of the water: for though the air at the bottom is necessarily loaded with moisture, yet in ri-sing to the height of 4 or 5 feet, so much of the water seperates & falls down, as to leave the air ~~sufficiently~~ seemingly of suf-ficient dryness. The vessel might be made as high as the pipe itself: nor would this larger size be of any inconvenience in regard to the blast, for as soon as it is filled with air of a certain density, the blast will continue of the same force as from a small vessel,

The joints should be well secured to prevent the escape of any air through them: the Stone for receiving the dash of water, should be placed near as much below the level of where the water runs off as the gage is expected to rise; & the pipe should reach as low as within 5 or 6 inches of the Stone. It would perhaps be of some advantage to have the Surface of the Stone a little concave, so as to occasion the watry drops to be rather dashed back-wards towards the Stream, than thrown upwards through the cavity of the Vessel. —— —

III Experiments of air passing down through pipes with falling water. —

Water running through a crane

In the running of water through a siphon or common crane, when the sucking pipe on the long leg of the crane was stoped, the water, as it issued from the extremity, filled the bore: on opening the sucking pipe, the column of water appeared less than the bore. — —

Judging that the motion of the water must be retarded in this last circumstance, I measured by a pendulum the times in which equal quantities of water run through the crane in both cases; & found in many trials, that the quantity which took the time of a hundred swings of the pendulum to run in when the sucking pipe was open, run in ninety-three, & sometimes ninety-two, when it was stopt.

As these differences seemed to proceed from air introduced in to the water through the lateral pipe; I tried to make this air sensible, by raising the vessel which received the water from the crane, & keeping the nose of the crane immersed in it. As often as the sucking pipe was opened, air bubbles arose in the water of the receiver, & fresh bubbles succeeded while it continued open; but so long as it was kept stopt, no air bubbles were seen. ——

To collect the air, a cask without a bottom was sunk 9 or 10 Inches in a tub of water, & the nose of the crane inserted into a hole made in the

top of the cask: into another hole in the top was fitted a small pipe for giving vent to the air; & within the cask was fixed an inverted Mortar for the stream to fall on. So long as water was kept running through the crane with the sucking pipe open, a sensible blast issued from the blowing pipe of the cask, & a burning coals exposed to it was excited in the same manner as by a common bellows: the sucking pipe being stopt, no blast was perceived, nor was any motion produced in the flame of a candle applied to the orifice. - - -
It appears therefore that water, running down through an upright pipe, & filling its bore, admits air to enter through a lateral pipe: that after this admission, the width of the column of water contracts, the introduced air occupying part of the cavity of the pipe; & that this air passes down on the outside of the water, or in a seperate column, not intermixed with it so as to render it frothy. - - -

Water descending through an Oblique pipe with lateral apertures. ——

I varied the foregoing experiment by taking, instead of the crane, a leaden pipe, about 10 f.t long & 3/4 of an inch bore. Several holes were made, at intervals, in the length of the pipe, & small tubes fixed into them like the sucking pipe of the crane. The pipe being laid a slope, its upper end was turned up perpendicularly, & a funnel fitted to it, which was supplied with water by a cock in the bottom of a reservoir: the other end of the pipe, which the water issued from, was inserted into the air vessel used in the ~~preceeding~~ experiment. - - -
The lateral tubes being stopt, & the cock so turned as to let the water run fast enough so as to keep the funnel always full, no air issued from the blowing pipe. On opening the tubes, a considerable blast was perceived; the water passed slower through the pipe, so that the same stream made the funnel run over; & on putting out some of the tubes, & looking in through the holes, the column of water was very visibly less than the bore of the pipe. The tubes being stopt again, the blast ceased, & the stream did no more than keep the funnel full. - - -

A small variation in the circumstances of this experiment made a very material difference in the effect. The supply of water having been diminished, so as to rise only a little way above the throat of the funnel, a pretty strong blast issued from the blowing pipe though all the lateral tubes were closely stopt; & when the tubes were open, instead of air passing in by them, a blast passed out from them, the air vessel in this case yeilding none; so that here the air must have been introduced at the top & passed down the funnel, & afterwards escaped where it first found a vent. To be further satisfied in this point, I repeated the experiment with a somewhat different apparatus, in the following manner. — — —

Water falling through a funnel

The glass receiver of an air pump, about two feet high, open at both ends, had its lower end immersed about seven inches in a vessel of water, and supported at a proper distance above the bottom for the free passage of the water under the edges. A brass plate being pressed close on the top, with leather between, a glass funnel, about 12 inches deep, and above half an inch diameter in the throat, was fixed into a hole in the plate; & into another hole was fitted a small blowing pipe. — A stopper being introduced into the funnel, till the water it was filled with had become perfectly quiet, & then cautiously removed, the water run in a stream, which falling into that in the receiver, produced air bubbles: but no blast issued from the pipe; & when the pipe was stopt, the water in the receiver did not sink lower than the level of that in the outer vessel, whereas, if any air had entered with the water, & been compressed in the receiver, it must have forced a proportional quantity of water out below. —

The funnel was then supplied from a pipe, by which the water was made to dash against one side of it. By this means the fluid received a spiral motion, and twirling round the funnel, left a large vacuity in the middle, reaching down sometimes to the funnels throat. The stream, as it run through, was also twisted; a sensible blast issued from the air pipe; when the pipe was stopt, the water in the receiver was forced lower & lower, & was soon driven entirely out,

abundance of air bubbles following it into the water in the outer vessel. —

When the funnel was kept entirely full: though the stream was directed as before against its sides, there were little marks of any air being carried down. And when the funnel was near empty, the effects were also considerable; the vacuity in the middle of the spiral circumvolutions of the water seeming to reach to the bottom, so as to suffer the air to escape upwards through the hollow column of water. - - - -

Water falling from a considerable height into a funnel with a pipe. —

A Leaden pipe, six feet high & an inch & a half diam., was inserted into an air vessel, with the water gage already described. Into the top of the pipe was fixed a tin funnel, whose throat fitted close to it; & into the funnel a stream of water was let fall, from a reservoir five feet above, in quantity sufficient to keep the funnel running over. This apparatus represents Mariotte's blowing machine described in the third article of the preceding section. - - —

The water, divided by the fall, pushed down abundance of air with it: a strong blast issued from the blowing pipe, & the gage rose high. On raising up the funnel a little, the stream that issued from it appeared all frothy: as often as the funnel was lifted up, the gage sunk, the air, which had been driven in by the dash of water, escaping between the funnel & pipe: on letting down the funnel close, the gage immediately rose again. — Instead of a fall of five feet, a stream was directed into the funnel from only about half that height. The gage still rose considerably, though not so high as before. —

It is observable, that in the circumstances of these experiments, a twirling motion communicated to the water in the funnel impeded the carrying down of the air, the gage always sinking on the water receiving such a motion, whereas, in those of the preceding articles, it seemed to be by the twirling of the water

that the air was pushed down. - - - -
It appears therefore that there are two ways of making air pass down with water through a funnel, one by directing the stream against the side of the funnel, the other by letting it fall from a great height: that in the one case the air enters between the spiral circumvolutions which the water forms in the funnel, & in the other between the drops into which a considerable part of it is reduced by the fall; that we cannot avail ourselves of both ways at once, the one impeding the effect of the other; & that in either case the air holes under the throat, so necessary in other machines, can have no place, as they give a vent to the air brought down from above. - - -

Water falling from a funnel through a pipe with air holes. —

The six-foot pipe, used in the foregoing experiment, continuing fitted into the air vessel, its upper orifice was widened, that the small end of a funnel-shaped copper pipe, of the same bore with the preceding funnel, might hang freely in it, without touching the sides. The funnel pipe reached up to the reservoir, & was kept always full, that the water might receive little or no air but at the vacuity between the nose of the funnel & the leaden pipe. -
In this situation, the quantity of air was much less than in the preceding: the water fell through the funnel in a stream not at all frothy, & the gage rose but a little way. I widened the aperture of the leaden pipe, to let in more air, but still the gage continued low. - - - - -
Into the orifice of the funnel I inserted a smaller pipe, whose diameter was one inch, & whose area was of consequence less than half of that of the leaden pipe. The blast was now strong, & the gage rose higher than when the water fell from an equal into the low funnel of the foregoing article. I tried funnels considerably

smaller, I found the gage still to rise high: but at last, with one of a quarter of an inch diat., it did not rise at all, & no blast could be perceived. - - —

One of the funnels which answered best being properly fixed, with two or three inches of its neck hanging free within the wider pipe, I made several variations in the manner of admitting the water & air, with a view to compare the effects of different ways of admission. The funnel being full, & gently supplied so as to keep the water in it as steady as possible, the height of the gage was marked: on giving a circular motion to the water, or letting it fall from a height, the gage always sunk, even a slight twirl or dash sensibly affecting its height. The space between the nose of the funnel & the pipe was stopt, so that no air could enter but at the top: the funnel being now full, & the water quiet, the gage scarcely rose at all; on twirling the water, it rose considerably, & when the water fell from a height, it rose further, though not so high as the standard mark. - - —

It appears therefore that there are two general methods in which water may be made to carry down air, one in which it receives the air at the top, & the other through lateral apertures; & that the circumstances, which contribute to the effect in one case, impede it in the other: that water, being at rest in a funnel, & then suffered to run through, carries little or no air with it; that when made to twirl round in the funnel, it carries a considerable quantity; & that when it falls from a height, so as to be in great part dashed into drops, it pushes down considerably more: That running through a pipe with lateral apertures, perpendicularly or obliquely, it receives air through the apertures, even when its motion is slow; that when the pipe is of equal bore throughout, the quantity of air thus received is not

great

great; but that, when the pipe is contracted to a certain degree in the part where the apertures are, the quantity of air is greater than that introduced through the funnel without air holes: That air brought down from the top of the pipe or funnel prevents the introduction of fresh air through the lateral holes, which in this case, instead of receiving more air, discharge that already received. - - -

Finding that the two general methods by which air is made to pass down with a stream of water, could not be united in one machine; & that the pipe & funnel, with apertures for the entrance of air about or under the throat of the funnel, have the greatest effect; I proceeded to examine the most proper form & disposition of them. - - -

IV. Experiments and Observations for regulating the Structure of the funnel and pipe

Experiments with funnels and pipes of different heights. - -

The water, as already observed, passing through the narrow throat of the funnel, is afterwards enlarged into a jet which fills the bore of a wider pipe. The quantity of air introduced appears to depend upon the degree of this enlargement, & on the quantity of water that runs through in a given time

The greater the height of the water above the narrow throat, the greater velocity will the jet receive, & the more it will be disposed to spread & be enlarged. The length of the pipes does not appear to be of such importance: it should seem sufficient if the pipe is of such length, that the pressure of water in it may be able to resist the compressed air in the air vessel, & that after part of its power has been spent in overcoming that

force, it may still have velocity enough left to run down as fast as it can be supplied from the funnel. In order to attain to some determinate proportions, the following trials were made. — —

A Leaden pipe, 7 feet high, & an inch & a half diameter, being fitted into an air vessel, as in the foregoing experiments, funnel-shaped pipes of different heights were supported over it, so as that the small end of the funnel might hang freely in the orifice of the leaden pipe, & leave space enough for the entrance of the air all round. For the greater security of the throat being of the same area in all the funnels, one & the same copper pipe served as a throat for them all: the funnels being formed by inserting this pipe into larger tapering ones of different heights. The funnels were always kept full, and the water conveyed into them as gently as possible, so as to produce no dashing or twirling motion. — — —

A funnel of one foot high had very little effect: the rising of the gage in the air vessel was very inconsiderable, & the stream of air in the blowing pipe was but just to be felt: on opening some holes made in the upright leaden pipe under the throat of the funnel, the jet of water appeared not ~~to~~ spread, but rather contracted, & did not fill the bore. With funnels of two & three feet high, the gage rose more, & the jet spread, though it did not appear to fill the pipe, till it had reached about half way down to the bottom. Funnels of 5 & 6 feet produced a strong blast, & kept the gage high, the jet filling the pipe before it had fallen a foot below the throat of the funnel. — —

On many repetitions & variations of these experiments, I have not observed that the jet spread sufficiently with less than a fall of 5 feet. With a fall of 64 inches, the gage rose more than five times as much as with one of 16 inches, though the quantity of water which run in the first case was only double to that in the latter, viz as $\sqrt{64} : \sqrt{16}$: from whence it is plain that the above differences do not depend entirely on the different quantities of water which run through funnels of different heights, but

in great part on its different velocity. Some other experiments seemed to confirm this point: for having used short funnels so much wider than the high ones, that the quantity of water discharged by the former was equal to or greater than that by the latter, the short never produced so strong a blast, or raised the gage so far, as the others. - -
Being satisfied of the advantage of having the funnel of very considerable height, I in like manner varied the length of the pipe. Having made a mark at the part where the gage rose to when the funnel was five feet, & the pipe seven, I added to the pipe about a foot more: the gage scarcely rose any further. A foot being cut off from it, the gage fell a little: two feet being cut off, it fell considerably; & the retrenchment of another foot made the machine of little effect, the gage sinking almost to the bottom, and the blowing pipe yielding but a weak current of air. The pipe thus reduced to four feet, was tried with a funnel of near 8 feet: in this case there was no blast at all. But with funnels less than its own height, as of two & three feet, it still raised the gage considerably. - - -
It appears from these experiments, that in most of the machines described in the preceding section, the lengths of the funnels & pipes are greatly disproportioned to one another, and consequently the water applied to disadvantage. Those of Dauphiny in France are particularly faulty in this respect, the funnel being scarcely three feet high, & the pipe twenty five or twenty six: with so small a height of water above the choak. I have never been able to make make the jet spread near to such a degree as it is said to do in the machines of Dauphiny, without particular contrivances for that purpose, which will be mentioned in the sequel of this paper. The Foix machine agrees best with my experiments: but as the funnels of the others are undoubtably too low, that of this seems to be rather too high. The effect appears to be the great-est, when the funnel is about two thirds of the length of the pipe.

Experiments of the disposition of the air holes. —

In the foregoing Experiments, the simplest & most obvious way of admit-ting the air was chosen, by leaving a space between the funnel & the pipe. The air pipes of the Machines of Foix & Languedoc answer the same end, carrying in the air above the surface of the jet of water. As the other Machines have the air holes under the jet, I tried what variations would result from this circumstance, and from making the apertures at different depths under the throat of the funnel. —

Into a pipe of 6 feet was fitted a funnel of 4 feet; & 6 inches below the orifice of the funnel, 4 holes were bored round ~~in~~ the pipe, sloping down from without inwards: 8 inches lower down, I made ano-ther row of holes; and at a like distance under these, a third & a fourth. To each hole was fitted a Stopper which exactly closed it. —

All the holes being stopt, the funnel was first hung free in the pipe, as in the former trials, and the height to which the water rose in the gage was marked. The funnel being then let down into the pipe, so as exactly to close it, the upper air holes were opened: the gage did not now rise so high as before. The upper air holes being stopt, & the second row opened, the gage continued at its last height. With the third row open, it rose rather higher than the first mark; and with the fourth it fell the lowest of all. — — —

The several entrances for the air were ~~then~~ opened, two by ~~and~~ two. With the space between the funnel & pipe, and the upper air holes, open, the gage did not rise so high as ~~with~~ the space only; & with the upper & second row of holes it continued at the same height. With the second & third, it rose considerably further, though not up to the first mark; and with the third & fourth, it fell a little below the preceeding height. In all these cases, where two rows of holes were open, the water manifestly did not fill the bore of the pipe at the upper holes; but spread so as to compleatly fill it by the time it had reached the lower ones, at which last, part of the water spirted out & carried some of the air with it. — —

In another pipe of the same size I made two sets of air holes, three inches apart, & the uppermost of them 12 inches from the orifice of the funnel. With the upper row open, & with both Rows open, the gage rose almost equally, being only a little lower in the latter circum-stance than the former: but with only the lower open, it sunk about one half. These being all stopt, and another set bored opposite to the orifice of the funnel, the gage rose as high as in the first case.—

These experiments, & several others I have made on the same subject, are not so conclusive as could be wished. They seem to show that it is more eligible, to have the entrances for the air in one horizontal plane, than in two planes above one another; & either above, or at some distance below the jet, than immediately under it: that they ought to be of greater magnitude than in some of the machines described in the first Section, particularly that of Leadhills, whose air holes, taken altogether, are not of half the area of the space in the pipe which the air has to fill. They ought at least to be of an equal, or rather of a double extent, that the air may enter the more freely.————

Experiments of the proportional bores of the funnel and pipe.—

We have already seen, that unless the throat of the ~~pipe~~ funnel is less than the pipe, the quantity of air carried down will be inconsiderable; & that by lessening it further than to a certain point, the effect is also diminished or destroyed. To hit this precise point is not perhaps possible; and the point which is the most perfect proportion for one height of water, cannot be so for any other, an increase of the pressure disposing the jet to spread more & fill a larger bore.—

It appears from some experiments already mentioned, that when the whole height of the fall of water is 15 feet, the height of the pipe ought to be 9 feet, & that of the funnel six. This being as low a fall as these kinds of machines have been generally erected for, & as high a one as is generally to be expected in this country, I

made several trials for adjusting the proportions to those heights; using for the funnel a tapering copper pipe, into the lower end of which were occasionally inserted smaller pipes of different bores. — By trying several of these funnels, we came to certain sizes, which could not be much increased or diminished, without diminishing the effect of the Machine; but if there is, in this respect, any exact standard, our experiments did not discover it. There are so many circumstances, as we have already seen, which influence the effect, that it is very difficult to judge, when the differences are small, how far they depend on any particular one. When the area of the orifice of the pipe was from four to five times greater than that of the funnel, the differences in the height of the gage were not very considerable: the due proportions seems to lye within these bounds, and perhaps nearer to the latter than the former; for when the funnel was only about a sixth part of the area of the pipe, the gag stood rather higher than when it was a third part, from whence the proportions should be as one to somewhat more than four & a half. — ——

Experiments of dividing the stream so as to increase its effect, and render less water sufficient. ——

As the effect of these kinds of machines depends on the waters being spread & divided, & the air, which comes in to fill the interstices between the little streams or drops which compose the jet, being pushed down with velocity by the succeeding water: I have endeavoured to divide the stream, more effectually than is done in the common machines, & with little or no diminution of its velocity, by varying the form of the aperture of the funnel. —

On the orifice of the funnel I fitted a perforated tin plate, like the nose of a watering pot, but with the holes larger, & of a triangular form: this form was chosen on account of its great surface, water, passing through a triangular aperture, having about a third part more surface than through a circular one of equal area: some more

holes were made round the sides, in such positions, that the streams issuing from the higher holes, might no where fall upon or coincide with those from the lower ones, but that the water might be uniformly dispersed through the whole cavity of the pipe. By this division of the water it was made to fill a much larger bore than otherwise, & to produce as great an effect as the full quantity of water which the same pipe would otherwise have required; insomuch that quantities of water which had little effect in the common way of application, were by this contrivance made to yeild a strong blast. - - - -

This method is accompanied with an inconvenience, which often showed itself in the course of the experiments, and which must be more considerable in the continued working of the machine. After it had acted vigourously for some time, its action frequently abated of a sudden: the blast from the blowing pipe grew weak, & the gage sunk: sometimes its force increased again in a little while, but for the most part it contimed to diminish more & more. The cause was discovered to be bits of leaves & other like matters which the water had carried into the funnel, & which had in part stopt up the small apertures. The remedy was obvious, letting the water pass from the reservoir through a wire sieve whose holes were much finer than those in the nose of the funnel; & doubtless an expedient of the the same kind would prove effectual for the largest machines. It is in all cases adviseable to have the water pass through a grating before it enters the funnel; even the common large apertures being sometimes choaked up by matters which the stream brings along with it. Where scantiness of water, or want of so high a fall as is commonly required, persuade to to this contrivance for procuring a more effectual division of it, and for augmenting its power with its surface, two or three grating

or perforated plates, with apertures of different sizes, will be necessary: one with very fine holes, much smaller than those of the cullender, that nothing may pass through the former which can be in danger of sticking in the latter: another with larger apertures, for detaining weeds, and such other matters as would soon obstruct the finer strainer. ——

I have tried other methods of producing this dispersion of the water, by making the throat of the funnels of different figures; but with little success. Whether the throat was made converging or diverging, in greater or less degrees, there did not appear to be any material difference in the effect of the machine. I introduced into the funnel a cylindrical core, which was fixed in the middle, by means of pins projecting from it, so as to leave a circular aperture all round it; and this core was sometimes solid, & sometimes a pipe which reached above the funnel & carried down air into the middle of the jet below: but no other difference was observed in either case than what arose from the necessary diminution of the quantity of water. It is probable indeed, that by duely proportioning the core to the funnel, & the width of the pipe to the sheet of water falling round the core, the effect, by this division of the stream, would be made greater than an equal quantity of water would produce when falling in one column; though the increase, obtainable by this method, did not promise to be considerable enough to deserve the troublesome investigation of the proportions. One trial however, depending partly on this principle, appeared of some importance to be made. As the water machine of S^t. Pierre is said to have two apertures in the bottom of the funnel, whose streams, as they issue out, cross one another and are dashed into drops, I tried to answer this intention, by using for the funnel a wooden trunk, with two of its sides sloping downwards so as to leave a long narrow

aperture between them: in the middle of this aperture, & parallel to the inclined sides, was placed a wedge of the same slope with the sides of the funnel, that the water might pass out in two sheets directed towards one another. ---

The funnel was at top about 8 inches square: its width at bottom $7\frac{8}{10}$ inches by $1\frac{9}{10}$. The wedge, dropt into it, entirely stopt the lower aperture, and had its thin edge hanging down considerably below: slips of wood of different thicknesses fastened on the wedge occasionally, two on each side, prevented its falling down so far, and procured spaces of different widths between it & the sides of the funnel; so that the water could be reduced at pleasure into two sheets, $7\frac{8}{10}$ wide, & from less than a quarter of an inch to three quarters of an inch thick; the partition in the middle reaching in all cases lower down than that which confined them on the sides, that the might not unite into one upon their discharge from the throat. Along the sloping sides of the funnel were two air pipes, of the same breadth with them, & about an inch & a half wide; so at the bottom there were three oblong rectangular apertures, the middle one, with a wedge in it, for the water, & the two lateral ones for air: the outsides were continued about 9½ inches below these apertures, so as to form a large cavity for the water to spread in,

The funnel, above the throat, was somewhat more than three feet: on the top was fitted a wooden pipe, nearly of the same width with it, & four feet Eight inches high. The top of this pipe passed up through a rectangular cystern, nearly 168 inches in length & 9 6 in width, & which consequently contained about 37 on every inch in depth. For admitting the water, two holes were made in two opposite sides of the pipe, about 10 inches high, with two sliders fitted to them, for occasionally varying their heights & consequently the quantity of water received. On the

outside of each hole was fixed an iron plate, perforated with numerous small holes, to keep back such matters as might choak up the throat: that the holes might be sufficient to allow water enough to pass in, the Strainer was made wider than the aperture of the pipe, and bent to a semi-cylindrical form

To the bottom of the funnel, enlarged as above mentioned, was fitted a pipe six feet high, & in width 4 inches by 7½. The lower end of this pipe was inserted into the head of a large cask without a bottom, which was set in a tub above three feet deep, with three supports under the lower edge of the cask to procure a space between it & the bottom of the tub for the water to pass freely off. About 8 inches under the orifice of the pipe, a round board, for the water to fall on, was hung by three cords, which passed up through the head of the cask & were secured by pegs. At one side, a tin vessel full of water was supported in the same manner; & through a faucet, over the middle of this vessel, was inserted a glass tube 3 4 inches long. At the other side was the blast pipe, about ¾ of an Inch diar. — —

The Machine being thus prepared, we proceeded to the trial of it, expecting that the two Streams, from their sloping direction towards one another, would cross & be dashed into drops, & carry down abundance of air. But in the effect we were greatly disappointed: the blast was weak, & the gage rose to no considerable height, whether the wedge was dropt down or drawn up, so as to suffer the water to pass in less or greater quantity, in thin or in thick sheets: in continued trials & variations of the apertures for 3 or 4 days, the gage was

not

once observed to rise so high as 10 inches. A good deal of air indeed escaped through the junctures of the pipe & air vessel, but not near enough to make up the expected quantity. –

The wedge answering so ill, it was laid aside; & in its place was introduced a leaden vessel, of the same shape with the funnels throat, & of such a size, as to rest against the sides of its aperture by its upper edge, & hang 6 or 7 inches down in the wider part of the pipe: in the sides & bottom of this vessel were made several holes, about 1/10 of an inch diar. With this alteration I had the pleasure to find, that though air rushed out from the joints even more plentifully than before, yet the blast from the blowing pipe was strong, & the water in the gage pipe rose to the top & ran over. –

I tried to measure the quantity of water necessary for producing this effect for a certain time. The reservoir being filled to the depth of 14 inches, the gage rose as before, & continued high for four minutes; after which it began to sink fast, the water in the reservoir having then become too low to keep the pipe full, though it continued to run for a considerable time longer. From the dimensions of the reservoir already mentioned it will appear, that if all the water had run out in four minutes it would have amounted to near two hundred gallons in one minute; but at least a fourth of it remained after that period, so that the expence could not exceed 150 Gallons ℔ minute. We could not expect any great accuracy in this determination, because as the height of the water continually decreased in the reservoir, its velocity likewise decreased, so that if a due quantity run in

the last minute, a superfluous must have risen in the first.
The leaden cullender being taken out, & the whole throat left vacant for the stream, the gage still rose to the top; but the expence of water was now double of what it was before. -
These trials though not carried to such a length as I could have wished, satisfied me, & those who assisted at them, that much more air is to be obtained, by dividing the stream by means of a cullender, than by any other methods that have been tried; & that with such a machine as is above described, a stream of 150 Gallons at most in a minute is sufficient to produce a continued blast, from a pipe of 3/4 of an inch bore, of such strength as to support a column of water of three feet or more. -
To afford as much assistance as possible to those who may be desirous of erecting machines of this kind, I shall here collect into one view the most material particulars which my experiments have discovered with regard to the perfection of their structure, and form from them the description of such a machine as promises to be the most effectual
The bottom of the reservoir of the water should be about 14 feet above the level of the ground: we need not be very solicitous about procuring a greater height, for though a greater would be of some advantage, yet this advantage appears to be much less considerable than has been commonly imagined. In the channel by which the water is conveyed, are to be placed gratings of different sizes, as already mentioned, & before the aperture a fine grating, which may be either a perforated iron plate or a wire sieve, to serve as strainers for keeping back such matters as would obstruct the apertures which the water is afterwards

to pass through. The stream should enter at one side, or be so managed, that the water in the reservoir or funnel may not be agitated by it, or put into a spiral motion, which our experiments have shewn to be very injurious.

In the bottom of the reservoir is to be made a round hole, for admitting the upper end of what we have hitherto called the funnel, but which may here be more conveniently a cylindrical pipe, of copper or of cast iron, 5 or 6 inches in the bore, 17 feet long. To the end of this pipe is to be fitted a cullender, about a foot long; with the holes triangular, of half an inch each side; & 6 or 7 strips from top to bottom, at equal distances, preserved without holes, for admitting air to pass down to the lower streams. All the holes should be directed downwards, that the streams may not be forcibly projected against the sides of the pipe which is to receive them, so as to have their velocity too much diminished.—

If there are six of the perforated spaces in the cullender, the number of holes in each may be twenty; so that the whole number will be 120. The side of each of the triangular holes being half an inch, the area of each will be the eight part of a square inch, & the sum of their areas will be 15 square inches. The quantity of water running through one aperture of such an area, at the depth of 7½ feet under the surface, comes out on calculation about 622 Gallons in a minute; but the real quantity will doubtless be much less than this, on account of the great friction of the water in passing thro' a number of small holes, & of the resistance of the air, which increases in a very high ratio according to the increase of velocity & enlargement of the surface: it is in part to make up for these retardations, that the pipe is directed to be made

so high. The surface of the water is here above 13 times greater than if it had passed all through one circular aperture. —
Both the pipe & the cullender should have a flange or rim round their orifices, & be secured to one another by screws passing through the rims of both, with a plate of lead between them to make the juncture tight, as commonly practised in joining iron pipes for water works. This way of joining them admits the cullender to be taken off & cleand; when a diminution of the effect of the Machine shews the holes to be choaked up, which however, it is apprehended, will seldom, if ever, happen. —
As the holes will permit more water to run through, than may at all times be wanted, it is proper to have some contrivance for occasionally closing a part of them. This may be effected by means of a thin copper pipe, open at both ends, as high as the cullender, & of such width as just to drop into it. It will be easily conceived, that when this register is let entirely down, the lateral holes will be covered, & the water admitted only to those in the bottom; & that by raising it further & further, more & more of the lateral holes will be uncovered. The register is to be hung by a wire to a cross barr over the reservoir, by which it may be raised or lowered; & a scale or divided board may be adjusted against the upper part of the wire, for shewing the height of the register, or the number of holes closed by it. —
The most comodious & effectual way of admitting air to the water appears to be that of our first experiments, viz hanging the throat of the funnel, in this case the cullender, within the wider receiving pipe, for by this means the air is admitted freely & uniformly all round. This last pipe should likewise be of iron or copper 12 inches in diar., & spread out at top to the width of

16 or 18 inches, that a large space may be left round the cullender: this space should reach three or four inches above the uppermost perforations of the cullender to prevent any of the water from being dashed over the top. ——

A pit is to be sunk in the ground, not less than six feet deep. In this is to be placed an air vessel, made of wood lined with lead, without a bottom, 3 or 4 feet in width, & 10 or 11 deep or high. The vessel should be supported on feet, of a proper strength, with sufficient spaces between them for the water to pass freely out: this way is preferable to the common one of placing the lower edge of the vessel on the bottom of the pit, & cutting an aperture in the side, because the height of the aperture is so much taken off from that of the vessel. The reservoir being 14 feet above ground, & the upper pipe & cullender reaching down 8 feet, only six feet remain below the cullender; so that the air vessel, having six feet sunk in the ground, will reach nearly up to the cullender, & almost the whole height of the under most pipe will be included within the vessel. This pipe may be above 9 feet long, 3 feet or more of it going down into the pit; which 3 feet are here an entire gain in the heights of the fall, for the pipe in the other machines comes at most no lower than the level of the ground where the water runs off on the outside. This height is gained, in virtue of the compressed air in the vessel pushing down the water below, as already shown in the second article of this section: it may be always as great as the height to which the water is intended to rise in the gage. At the distance of 5 or 6 inches under the orifice of the pipe is to be placed the concave iron plate or stone for the water to fall on. In the top of the air vessel is to be fixed the gage & the blowing pipe —

Such is the general construction of the blowing Machine, which promises to be particularly usefull in cases where water is scarce or where the want of a natural fall renders it necessary to raise, by very expensive means, the great quantities requisite for working the common bellows. It is presumed, that one of these Machines will be sufficient for the Iron forge, & for sundry other purposes where the quantity of air is not required to be very great; that it will be less ~~expensive~~ expensive, on account of the durabili-ty of its materials, & the simplicity of its structure, than any kind of bellows no in use; & what is of principal importance, that much less water will serve for working of it. In cases where one of the Machines cannot supply air enough, as for the large Iron smelt-ting furnace, two pipes may be used, both fed by one reservoir, & entering into one air vessel, as practised in some of the instruments described in the first Section. The using of two pipes appears more eligible than enlarging the bore of one; for air cannot be so freely introduced into a large body of water, though divided into streams by the cullender, as into two smaller ones of equal quantity. — — It may be observed, that the blast will be stronger in a dense state of the atmosphere, than when it is more rare or expanded, a greater quantity of air being then introduced under an equal volume. If therefore the quantity of water has been adjusted so as to raise the gage to a ~~proper~~ height when the air is light, it will frequently happen that the same quantity of water will raise it higher, & consequently, if no greater height is re-quired, that a part of the water may be saved. As the gage of our Machine discovers by inspection these variations in its effect, the regulator affords convenient means of regulating its power, & increasing or diminishing the quantity of water.

I have received an account, from a worthy correspondent in Swisserland, of a machine he has constructed for a smelting furnace according to the forgoing directions: he says, it has so much the advantage of all other kinds of bellows, that it deserves to be introduced universally wherever the situation of the place will permit. The only inconvenience he finds in it is, that the cullender & gratings are liable to be stopt up by leaves, &c. With regard to the cullender, the obstruction may be obviated by enlarging the holes. The gratings ought to be of a large surface: the wire grating in the cistern on the Top may be a cylinder nearly as large as the cystern will receive, for if it is no more than sufficient to cover the mouth of the pipe, it will doubtless be soon choaked up: when so much of the cylinder becomes stopt, that the water has no longer a free passage through, it may be lifted up & cleaned, another being placed in the room of it, without the trouble of turning off the water, or interrupting the going of the machine. The gratings here can be liable to no other inconveniencies, than those which are common in other water machines, Mills, aqueducts &c. —

Some further improvements have occurred in the construction of these machines, by which they may be made effectual in cases where the quantity or fall of water would otherwise be insufficient. —

Of constructing blowing machines with falls of water of great height

Where the height of the fall is great, the quantity of water is usually small; & in all the ways of application that have hitherto been contrived, the height will by no means make amends for the deficiency in quantity —

In the common construction of those machines, where the upper pipe or funnel is no more than 3, 4, or 5 feet high; though the fall should be such as to admit of the lower pipe being 30, 40 or more, it does not appear that any material advantage could result from such a height. For, as the air is admitted into the water only at the top of this long pipe, it cannot, I think, be supposed, that the quantity admitted will be greater for the length of the passage under the place of its admission. Water indeed has been found by Mariotte to run faster, through an upright long pipe, than through a short one: a quantity of water was 45" in running through a pipe three feet long, was discharged in 37", or near a sixth part less time, through a pipe of the same bore & double length; so that as more water passes successively through the long pipe than through a short one, in equal times, more air must also be carried down by it. But in the case which we are here considering, no benefit can be expected on this principle; for as the supply of water is supposed to be limited, the bore of the pipe must necessarily be made less, in proportion to the increase which its length may produce in velocity. If the lower pipe is of such height, that the watery column it contains may sufficiently resist the force of the compressed air in the air vessel, it should seem that any further addition to its height could be of no manner of use. — —

We have seen, in the foregoing part of this essay, that it would be more advisable, in such cases, to shorten the lower pipe, and to lengthen the upper one: by this means the water

acquiring greater velocity at the place of its discharge from the upper pipe into the lower, is enabled to divide or spread more, & thus to receive more air into its interstices. The advantage, thus obtained, does not however increase in so great a proportion as the height does. From an experiment related in page 310 it appears, that by increasing the height fourfold, the effect was not increased three-fold; & this even in small heights, where the effect is much more influenced by a variation of the heights than in great ones. ——

The observations already mentioned point out the means of availing ourselves more advantageously of high falls: so as to produce always with certainty, from a fall of a double or treble height, a double or treble effect if the quantity of water be the same; or an equal effect, with one half or one third the quantity of water. ...
Experiments have convinced me, that a fall of 14 feet is more than sufficient for compressing the air to such a degree, as to be able to sustain the gage at the height of 4 feet; or to raise, on an opening of a square inch, a weight of about a pound & three quarters avoirdupois, or about two pounds troy; a compressure, which is apprehended to be as great as their will in general be occasion for. Where we have plenty of water, with such a fall, we can drive in air, with this force, in any quantity: for if one machine, with a certain portion of the stream, produces a continued blast of this strength through a pipe of a certain bore, as an inch or three fourths of an inch; it is evident, that the quantity of air may be doubled, trebled, &c. at pleasure, without diminishing the compressure or force of the blast, by adding another & another machine, till all the stream is employed. It is plain, in like

manner, the same advantage may be received from high falls, by placing one machine over another; that after the water has performed its office in falling through one machine, it is still capable of exerting the same action in another & another machine, so long as equal spaces remain for it to fall through; so that the total effect must be the same, as if a quantity of water, sufficient for working all the machines, came at first in one stream.

A fall thus divided into two machines is reprepresented in the middle of the annexed plate. In the lower machine, whose air vessel is sunk to a considerable depth in a pit made in the ground, the water is forced up in the pit, on the outside of the vessel, 4 feet higher than the surface of the water within the vessel, or of the stone on which the water dashes, called by the workmen the dash-board. The air vessel of the upper machine having an additional part at one side, which performs the same office as the pit, the water is in like manner forced up to the same height in this outer part; which outer vessel serving as a reservoir for the machine under it, the water begins to act in this lower machine 4 feet higher up than the dash board of the first. Whatever number of machines the fall will admit off, the case is the same in them all: though in each of them the water falls 18 feet, yet as it is pressed up again 4 feet for the succeeding machine, one machine takes up but 14 feet of the real fall. -

The outer vessel, & its communication with the air vessel, may be conveniently formed by an upright partition in the air vessel itself, not reaching quite to the bottom. The outer division may be open at top, & needs not be so high as the close air vessel; it is sufficient if it reaches a little more than four feet above the level of the dash board, the water, which it is designed to receive, not rising higher than this. In other respects, the structure of

these machines agrees entirely with that of the single ones already described. It must be observed only that the cullenders of the lower machines should be, as nearly as possible, of the same dimensions with those of the upper ones. For if they are of smaller bores, they will not admit of all the water which passes through the upper ones, so that part of it must run to waste: if they are larger, the water will pass off too fast, without producing its due effect. The regulators, formerly are here particularly useful, affording ready means of increasing or diminishing the apertures occasionally while the machines are at work. --

Of blowing machines with low falls of water

The dimensions hitherto given are such as appear the most advantageous. Much lower falls, however, than those which the foregoing machines are calculated for, as 10, 8, or perhaps 7 feet, may be made to afford a strong blast. To produce such a compressure of air in the air vessel, as to raise the gage four feet, a fall of about six feet is necessary for the lower pipes. If the upper pipe is only about 1½ feet ~~high~~ or 2 feet, the water when divided by means of the cullender, will carry down a certain quantity of air; & though the quantity from an equal stream of water will not be so great as when the fall is higher, yet, as there are many parts of the Kingdom, large bodies of water running with such a descent, the deficiency may be compensated, as already taken notice, by enlarging or multiplying the machines. -- For many purposes still less falls will suffice, the smiths bellows, as we have formerly seen, raises the gage only about the inches; & such a compressure, it is presumed, may be gained

from a fall of 5 feet or less. Small falls may be applied also to another purpose, of no little importance, the ventilation of Mines & Coal pits, or the driving in of fresh air, in the room of that, which the mineral vapours have rendered unwholesome or pernicious. –

In all these Machines it must be observed, that the height of the column of water, falling through the pipe, determines, not the actual force of the blast, but the greatest force that can be given it in that Machine; that the height of the gage is always the measure of the actual force; that this depends on the width of the pipe through which the air is discharged from the air vessel, & may be diminished, or increased in any degree up to the greatest that the column of water can resist, by widening or narrowing the aperture of the pipe; that different machines will give blasts of equal force through pipes of greater or less width, according to the greater or less quantity of air which the water carries down with it; & that therefore the size of the blast pipe must be adjusted by trial for each particular Machine. –

The distance of the dash board under the pipe may likewise admit of ~~some~~ variation, & requires to be regulated according to the size of the pipe. In some of the common Machines, this distance is 3 or 4 feet or more; but so large a space is apparently a disadvantage; for so much of it, as is more than sufficient for the free passing off of the water, is entirely useless being, in effect, so much taken off from the height of the fall. The distance of six inches, laid down in the foregoing Machines, is designed for a circular pipe of 12 inches diam; in which case, the area,

by which the water is discharged all round, is just double to the Area of the pipe, & consequently more than large enough for letting the water off without impediment —

Explanation of the plate

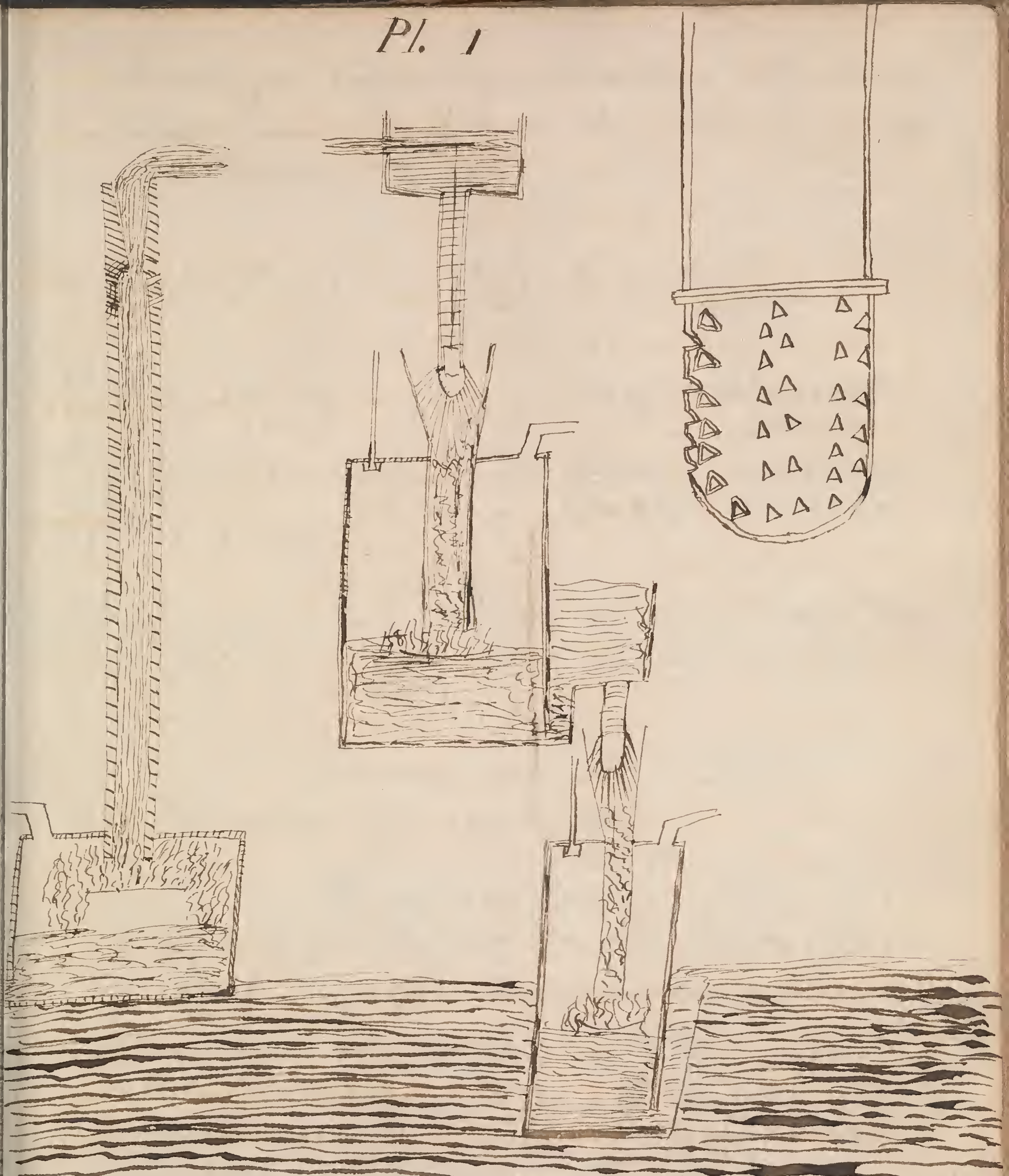
The two blowing Machines, represented on the plate, are both drawn to one Scale, that the eye may judge more readily of their comparative heights & dimensions. The supports of the reservoirs, &c are not expressed, that the essential parts may be more distinct.

The Machine on the left hand is that of Dauphiny, described aforemade, with a fall of about 30 feet. The other is a natural fall of 28 feet, formed into two artificial ones. this double Machines, though somewhat lower than the other, may be presumed to have twice its effect, in virtue of the division; besides the advantage of the more free admission of Air, & the spreading of the Stream through a pipe of a much larger bore, by which it is enabled to carry down in its interstices a much greater quantity of Air. The dotted lines, in the upper reservoir, represent a cylindrical grating of Iron Wire, to keep back weeds &c. The division of the air-vessel, & the course of the water from the upper Machine to the lower, are apparent from the figure

On the right hand is a perspective view of the cylinder, screwed to the upper pipe, drawn to a larger Scale, to shew the disposition of the holes. The holes may be made wider than formerly proposed, as an inch each side, to prevent any danger of their being choaked up. — — — —

FINIS.

Pl. 1



Theorems for Computing Logarithms. By the Rev. John Hellins: communicated to the Royal Society by the Rev. Nevil Maskelyne D.D. F.R.S.

Theorem I.

The Log. of $\frac{p+q}{p} = 2 \times$ log. of $\frac{2p+2q}{2p+q}$ + log. of $\frac{\overline{2p+q}^2}{\overline{2p+q}^2 - q^2}$

Demonstration

$\frac{\overline{2p+q}^3}{\overline{2p+q}^2 - q^2} \times \frac{\overline{2p+2q}^2}{\overline{2p+q}^2} = \frac{1}{\overline{2p+q}^2 - q^2} \times \frac{\overline{2p+2q}^2}{1} = \frac{4p^2+8pq+4q^2}{4p^2+4pq} =$ $\frac{p^2+2pq+q^2}{p \times \overline{p+q}} = \frac{p+q}{p}$: consequently, log $\frac{\overline{2p+q}^2}{\overline{2p+q}^2 - q^2} + 2$ l. $\frac{2p+2q}{2p+q} =$ log. $\frac{p+q}{p}$. Q.E.D. —

Corrollary —

If $q=1$, & we write n for p the theorem becomes log. $\frac{n+1}{n} =$ 2 log $\frac{2n+2}{2n+1}$ + log. $\frac{\overline{2n+1}^2}{\overline{2n+1}^2 - 1}$, which expression perhaps is of more frequent use than before. —

Theorem II. —

Log $\frac{p+q}{p} = 2$ log. $\frac{2p+q}{2p}$ − log $\frac{\overline{2p+q}^2}{\overline{2p+q}^2 - q^2}$ — —

Demonstration

$\frac{2p+2q}{2p+q} \times \frac{2p+q}{2p} = \frac{2p+2q}{2p} = \frac{p+q}{p}$: therefore, log $\frac{p+q}{p} =$ log. $\frac{2p+2q}{2p+q}$ + log $\frac{2p+q}{2p}$. But it has been proved above that log. $\frac{p+q}{p} = 2$ log. $\frac{2p+2q}{2p+q}$ + log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2 - q^2}$: If now we take this equation from twice the last there will remain 2 log. $\frac{p+q}{p}$ − log $\frac{p+q}{p} =$ 2log. $\frac{2p+2q}{2p+q}$ + 2 log $\frac{2p+q}{2p}$ − 2log. $\frac{2p+2q}{2p+q}$ − log $\frac{\overline{2p+q}^2}{\overline{2p+q}^2 - q^2}$: that is, log $\frac{p+q}{p} = 2$ log. $\frac{2p+q}{2p}$ − log. $\frac{\overline{2p+q}^3}{\overline{2p+q}^2 - q^2}$ Q.E.D. —

Corollary.

Putting $q=1$, & $n=p$ as above we have $\log\frac{n+1}{n} = 2\log\frac{2n+1}{2n} - \log\frac{\overline{2n+1}^2}{\overline{2n+1}^2-1}$ — I shall now set down some examples of the use of these theorems beginning with Theorem 1. —

The first example of the utility of this Theorem may be in computing the logarithm of the number 2

It is well known to Mathematicians, that the computation of this logarithm was formerly a very laborious task: and although the work may be much shortened by help of the converging series invented by the illustrious Sir Isaac Newton, still the logarithm of 2 has not been directly computed without many figures by any theorem I have yet seen. The easiest computation of it that has come to my hands is in page 44 of the late ingenious Mr. Thomas Simpsons pamphlett on Trigonometry & logarithms. His series consists of the powers of $\frac{1}{3}$

If now we put $n=1$ in the theorem ($\log\frac{n+1}{n} = 2\log\frac{2n+2}{2n+1} + \log\frac{\overline{2n+1}^2}{\overline{2n+1}^2-1}$) we shall have $\log\frac{2}{1} = 2\log\frac{4}{3} + \log\frac{9}{8}$. Here thus the fractions whose odd powers are to be used, are $\frac{1}{7}$ & $\frac{1}{17}$; consequently in the series formed from $\frac{1}{7}$, about $\frac{1}{2}$ of the number of terms taken by Mr. Simpson will give the result true to as many places of figures as his; & from the fraction $\frac{1}{17}$, much fewer terms will suffice. To show how fast these series converge I will set down of each [illegible] terms, enough to give the logarithm of 2 true to ten places of figures. —

The odd powers of $\frac{1}{7}$ divided by their respective indices

1st	0.14285714286
3d	0.00097181730
5th	0.00001189980
7th	0.00000017347
9th	0.00000000275
11th	0.00000000004
The Sum	0.14384103622 is $\frac{1}{2}$ l. of $\frac{4}{3}$
	4
	0.57536414488 twice l. $\frac{4}{3}$

The odd powers of $\frac{1}{17}$ divided by their respective indices

1st	0.05882352941
3d	0.00006784721
5th	0.00000014086
7th	0.00000000035
	0.05889151783 is $\frac{1}{2}$ l. of $\frac{9}{8}$
	2
Log. $\frac{9}{8}$	0.11778303566
2 Log. $\frac{4}{3}$	.57536414488
Log 2	.69314718054

But it is obvious that this operation gives not only the logarithm of 2 but that of 3 also; for the logarithm of 4 being given from that of 2 and the logarithm of $\frac{4}{3}$ computed above, the logarithm of 3 is had, being = log of 4 − log. of $\frac{4}{3}$

Log. of 4	1.38629436108
Log. of $\frac{4}{3}$	0.28768207244
Log. of 3	1.09861228864

Other examples of the use of these theorems in shewing how easily the logarithms of great fractions are derived from those of small ones. ——

If the logarithm of $\frac{64}{63}$ were given, or computed, we may very easily find the logarithm of $\frac{32}{31}$: for (by the theorem) 2 log. $\frac{64}{63}$ + log $\frac{\overline{63}|^2}{\overline{63}|^2-1}$ = log. $\frac{32}{31}$. Here the fraction whose odd powers are to be used in the series is $\frac{1}{7937}$, and the very first term of it, will give the logarithm true to 12 places of figures.

Again, if the logarithm of $\frac{16}{15}$ were to be computed from that of $\frac{32}{31}$ found above, we should have 2 log. $\frac{32}{31}$ + log $\frac{\overline{31}|^2}{\overline{31}|^2-1}$ = log $\frac{16}{15}$. Here the fraction to be used in the series is $\frac{1}{1921}$, the first term of which will give the logarithm true to ten places of figures. —

In like manner, from the logarithm of $\frac{16}{15}$ we may find

that of $\frac{8}{7}$; from the logarithm of $\frac{8}{7}$ that of $\frac{4}{3}$; & from the logarithm of $\frac{4}{3}$ that of $\frac{2}{1}$ as is done above. The respective fractions for the series will be $\frac{1}{449}$, $\frac{1}{97}$, & $\frac{1}{17}$. —

Thus far the fractions I have taken have even numbers for their numerators; let us now take one whose numerator is an odd number $\frac{9}{8}$. Here $n = 8\frac{1}{2}$ $\log.\frac{9}{7}\left(\frac{4\frac{1}{2}}{3\frac{1}{2}}\right) = 2\log.\frac{9}{8} + \log.\frac{64}{63}$; and the fraction whose odd powers are to be used is $\frac{1}{127}$. Hence we have the log. of $\frac{8}{7}$ (for $\frac{9}{7} \div \frac{9}{8} = \frac{8}{7}$) & may proceed to find the logarithm of 2 as above. But the logarithm of $\frac{8}{7}$ may be directly derived from the equation thus: the equation in other terms is, $\log. 9 - \log. 7 = 2\log. 9 - 2\log. 8 + \log.\frac{64}{63}$; then, by transposition, $\log. 8 - \log. 7 = \log. 9 - \log. 8 + \log.\frac{64}{63}$, or $\log.\frac{8}{7} = \log.\frac{9}{8} + \log.\frac{64}{63}$. — —

But when the numerator of the fraction, whose logarithm is given, is odd, theorem 2nd is more commodious. For taking $\frac{9}{8}$ as before, we have $2\log.\frac{9}{8} - \log.\frac{81}{80} = \log.\frac{5}{4}$, where the fraction to be involved is $\frac{1}{161}$. Again $2\log\frac{5}{4} - \log\frac{25}{24} = \log.\frac{3}{2}$, where the fraction is $\frac{1}{49}$. And $2\log\frac{3}{1\cdot 2} - \log.\frac{9}{8} = \log.\frac{2}{1}$, where we have only to take the difference of logarithms, as the logarithm of $\frac{9}{8}$ as well as that of $\frac{3}{2}$ is given. —

All the above calculations are of hyperbolic logarithms; but the same theorems hold good for Mr. Briggs's or any other. I will give an example in the computation of Briggs logarithms of 7 from others already known

Let the logarithms of 100, 99, and 50, be given then (by theorem 1) $2\log.\frac{100}{99} + \frac{\overline{99}|^2}{\overline{99}|^2 - 1} = \log.\frac{50}{49}$, or $\log.\frac{100}{99} + \frac{1}{2}\log.\frac{\overline{99}|^2}{\overline{99}|^2 - 1} = \frac{1}{2}\log.\frac{50}{49}$; & then $\frac{1}{2}\log. 50 - \frac{1}{2}\log.\frac{50}{49} = \frac{1}{2}\log. 49 = \log. 7$. — —

$$\text{Log. of } \frac{100}{99} \text{ ------------ } 0.00436480540245$$

$$\tfrac{1}{2}\text{ Log. of } \frac{\overline{99}^2}{\overline{99}^2-1} \left(= \frac{0.43429\ \&c.}{19601}\right) \text{ ---- } \underline{0.00002215675128}$$

$$\tfrac{1}{2}\text{ Log. of } \frac{50}{49} \text{ ------------ } 0.00438696215373$$

$$\tfrac{1}{2}\text{ Log. of } 50 \text{ ------------ } \underline{0.84948500216801}$$

$$\text{Log. of } 7 \text{ ------------ } 0.84509804001428$$

Scholium

Neither the number 2, nor the fraction $\frac{64}{63}$ is chosen as the most advantageous to begin with in computing a table of logarithms but they are taken as some of the first that occurred, to show the use of these theorems. Perhaps there are other instances in which they would be shown to much more advantage; but I hope their use will appear from the few examples given. They may indeed be transformed so as to be more commodious in particular cases, and there may be some others derived from them, one or two of which I will here put down.

It is evident from theorem 1 & 2nd that 2 log. $\frac{2p+2q}{2p+q}$ + log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-q^2}$ = 2 log. $\frac{2p+q}{2p}$ − log $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-q^2}$: consequently 2 log. $\frac{2p+2q}{2p+q}$ + 2 log $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-q^2}$ = 2 log. $\frac{2p+q}{2p}$, or log. $\frac{2p+q}{2p}$ = log. $\frac{2p+2q}{2p+q}$ + log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-q^2}$ which may be called theorem 3^{d}.

Again, this equation may be thus expressed: log. $\overline{2p+q}$ − log. $2p$ = log. $2p+2q$ − log. $2p+q$ + log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-q^2}$; &, by transposition 2 log. $2p+q$ = log. $2p+2q$ + log. $2p$ + log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-q^2}$, which may be called theorem 4. & this is in effect one of the theorems given by D^r Halley, in Ph.^{l} Transactions, N^o 216, & which he says converges so fast that in his Oppinion nothing better was to be hoped. —

Dr. Dobsons Tables of evapouration at Liverpool

Table I

A comparative view of the evaporation, rain winds, and temperature of the air, during the year 1772. —

Months	Temp.	Winds	Evap.	Rain	Seasons
January	38	. . .	1.27	3.26	E 4.87
February	39	. . . .	1.25	2.35	R 7.23
March	44	. . .	2.35	1.62	T 40.
April	48	. .	2.53	1.85	E 11.40
May	57	. .	4.25	3.42	R 8.39
June	67	. .	4.62	3.12	T 57. -
July	70	. .	5.53	1.59	E 13.20
August	68	. .	5.35	3.65	R 11.29
September	62	.	2.32	6.05	T 66. -
October	60	. .	3.18	3.42	E 6.46
November	50	. . .	2.15	4.85	R 10.48
December	44	. .	1.13	2.21	T 51. —
	54 —		35.95	37.39	

Table II.

1773. —

Months	Temp.	Winds	Evap.	Rain	Seasons
January	44	. . . .	1.85	3.15	E 5.76
February	42½	. . .	1.23	2.37	R 6.17
March	50	. . .	2.76	0.65	T 45 —
April	54	. . . .	2.89	2.47	E 9.34
May	57	. .	2.79	4.56	R 8.35
June	64½	. .	2.66	1.42	T 58. —
July	67	. .	4.92	1.32	E 14.02
August	70	.	5.75	2.21	R 10.08
September	60	. .	3.35	6.55	T 65 —
October	55	. . .	2.79	4.57	E 5.49
November	47½	. . .	1.15	6.69	R 15.58
December	41½	. . .	1.55	4.32	T 48. —
.	54½		32.59	40.18	

D: Dobsons Tables of Annual

Tab. III. 1773

Months	Temp.	Winds	Evapor.	Rain	Seasons
January	44	. . . . .	1.85	3.15	E 5.74
February	42½	. . .	1.13	2.37	R 6.17
March	50	. . .	2.76	0.65	T 45.
April	54	. . .	2.89	2.47	E 9.34
May	57	. .	3.76	4.56	R 8.35
June	64½	. .	2.66	1.42	T 58 - -
July	67	. .	4.92	1.32	E 14.02
August	70	.	5.75	2.21	R 10.08
September	60	. .	3.35	6.55	T 65. -
October	55	. . . .	2.79	4.57	E 5.49
November	47½	. . .	1.15	6.69	R 15.58
December	41½	. . .	1.55	4.32	T 46. -
	54½		34.59	40.18	

Tab. IV. 1774. ——

Months	Temp.	Winds	Evaporat	Rain	Seasons
January	37	. . .	1.38	4.43	E 5.92
February	45½	. . . .	1.67	2.43	R 8.23
March	49⅔	. . .	2.87	1.38	T 44 -
April	54½	. . . .	4.56	2.23	E 12.39
May	59½	. . .	4.31	1.65	R 7.14
June	63	. .	3.52	3.26	T 59. -
July	66⅔	. . .	4.97	2.68	E 13.51
August	67	. .	4.52	2.36	R 10.56
September	61⅓	. .	4.02	5.52	T 65. -
October	57	: .	1.95	1.68	E 4.82
November	46⅓	. . .	1.12	2.69	R 6.00
December	41¾	. .	1.75	1.63	T 48½ -
	54		36.64	31.93	

Evaporation at Liverpool—
Table 4th 1775

Months	Temp.	Winds	Evaporat	Rain	Seasons
January	44½	. . .	1.51	3.21	E 7.10
February	49	. . . .	3.02	4.62	R 10.28
March	48½	. . .	2.57	2.45	T 47½.
April	57⅝	. . .	3.20	1.01	E 15.09
May	61	. . .	5.03	0.55	R 3.98
June	70½	. . .	6.86	2.12	T 63.–
July	68½	. .	5.03	5.31	E 12.50
August	66½	. .	4.42	4.26	R 12.57
September	65	. .	3.05	4.00	T 56½–
October	54½	. . .	2.12	7.01	E 5.27
November	45	. .	1.63	3.03	R 13.39
December	48½	. .	1.52	3.35	T 49⅓
	54		39.96	40.22	

The first column of the above tables points out the mean temperature of the Air at two o'Clock afternoon. The second the character of the month with respect to the winds, the number of dots expressing their strength; & to make this part tolerably accurate, daily observations on the winds were marked down, and the character of the month formed from a general survey of these Observations: our winds are westerly for near two thirds of the year. The third column points out the evaporation of each Month in inches & decimal parts of an inch. The fourth the depth of the rain during each month. And the fifth, the state of the seasons. E being prefixed to the evaporation of the whole three months, R to the rain, & T to the mean temperature.

It is to be observed that in making these experiments 251 grains were allowed for every cubic inch of water; & that three pounds and twelve ounces of water gave a depth of one inch on a circular area of 12 inches diam!.

If we take the medium of four years Observations it appears, that the annual evaporation at Liverpool amounts to 36.78 Inches.

The evaporation of the four summer months in a medium of four years was only 18.88 inches

Dr. Hales says the quantity of evaporation from the surface of the earth in hop grounds is 6.66 inches that is less than from water in the proportion of 36.78 : 6.66 or of 6 : 1 nearly

The medium quantity of rain that falls is 37.43 inches & therefore the rain is to evaporation as 37.43 : 36.78

Mr. Towneley of Towneley determines the depth of rain taken on a medium of 15 years in the neighbourhood of the hills which divide Lancashire from Yorkshire to be equal 41.516 but as his expt. were made ten yards above the surface of the ground which from some experiments I have made will make a difference of at least one third & therefore the ~~evaporation~~ Rain that falls will not be less than 55 inches

Description & use of a portable Wind Gage By Dr. James Lind, Physician at Edinr.

This simple instrument consists of two glass tubes AB, CD of five or six inches long (as in fig 1). Their bores, which are so much the better always for being equal, are each about $\frac{4}{10}$ths of an inch diamr. They are connected together like a siphon, by a small bent glass tube ab the bore of which is $\frac{1}{10}$th of an inch diamr. On the upper end of the leg AB there is a tube of latten brass, which is kneed or bent perpendicularly outwards, and has its mouth open towards F. On the other leg CD is a cover, with a round hole G in the upper part of it $\frac{2}{10}$ths of an inch diamr. This cover & the kneed tube are connected together by a slip of brass CD, which not only gives strength to the whole instrument, but also serves to hold the scale HI. The kneed tube & cover are fixed on with hard cement or sealing wax. To the same tube is soldered a piece of brass e, with a round hole in it, to receive the steel spindle KL, and at f there is just such another piece of brass soldered to the brass hoop gh, which surrounds both legs of the instrument. There is a small shoulder on the spindle at f, upon which the instrument rests, & a small nut at i to prevent it from being blown of the spindle by the wind. The whole instrument is easily turned round upon the spindle by the wind, so as always to present the mouth of the kneed tube towards it. The end of the spindle has

a screw on it; by which it may be screwed into the top of a post, or stand made on purpose. It has also a hole at L to admit a small lever for screwing it into the wood with more readiness & facility. A thin plate of brass k is soldered to the kneed tube, about half an inch above the round hole G, so as to prevent rain from falling into it. There is likewise a crooked tube AB (fig. 3, to be put on occasionally upon the mouth of the kneed tube K in order to prevent rain from being blown into the mouth of the wind-gage, when it is left out all nights, or exposed in the time of rain. The force or Momentum of the wind may be ascertained by the assistance of this instrument, by filling the tubes half full of water, & pushing the scale a little up or down, till 0 of the scale when the instrument is held up perpendicularly, be on a line with the surface of the water, in both legs of the wind-gage. The instrument being thus adjusted hold it up perpendicularly, and turning the mouth of the kneed tube towards the wind, observe how much the water is depressed by it in one leg, and how much it is raised in the other. The sum of the two is the height of a Column of water which the wind is capable of sustaining at that time; and every body that is opposed to that wind, will be pressed upon by a force equal to the weight of a Column of water, having its base equal to the surface that is opposed, & its height equal to the altitude of a Column of water sustained by the wind in the wind-gage. Hence the force of the wind upon any body where the surface opposed to it is known, may be easily found; & a ready comparison may be made betwixt

the strength of one gale of wind & that of another, by knowing the heights of the columns of water, which the different winds were capable of sustaining. The heights of the column in each leg will be equal provided the legs are of equal bores; but unequal if their bores are unequal. For suppose the legs equal, and the column of water the wind sustains to be three inches, the water in the leg, which the wind blows into will be depressed 1½ inch. below O, and raised just as much above it in the other leg. But if the bore of the leg which the wind blows into, be double that of the other, the water in that leg will be depressed only one inch, whilst it is raised twice as much, or two inches in the other; & vice versâ, if the same wind blow into the smaller leg, it will depress the water in it two inches, whilst it raises it only one inch in the other. The force of the wind may be likewise measured with this instrument, by filling it untill the water runs out at the hole G. For if we then hold it up to the wind as before, a quantity of water will be blown out; & if both legs of the instrument are of the same bore, the height of the column sustained, will be equal to double the column of water in either leg, or the sum of what is wanting in both legs. But if the legs are of unequal bores, neither of these will give the true height of the column of water which the wind sustained. But the true height may be obtained by the following formulae.

Suppose after a gale of wind, which had blown the water into one of the tubes from A to B (fig. 3) forcing it at the same time through the other tube out at E, the

surface of the water should be found standing at some level as DG, and it were required to know what was the height of the column EF or AB, which the wind sustained. In order to obtain which, it is only necessary to find the height of the columns DB or GF, which are constantly equal to each other: for either of these added to one of the equal columns AD. EG, will give the true height of the column of water which the wind sustained. —

Case I.

Let the diam. AC, EH, of the tubes be respectively represented by c, d; and let a = AD or EG, & x = DB or GF. Then it is evident, that the column DB is to the column EG as c^2x to d^2a. But these columns are equal. Therefore $c^2x = d^2a$; and consequently $x = \frac{d^2a}{c^2}$. —

Example

If the diam.s AC, EH, be respectively 10 & 1 and AD or EG = 3.96 inches x will = .0396 of an inch. For $d^2a = 1 \times 3.96 = 3.96$, which divided by $c^2 = 100$ gives $x = .0396$. —

Case II. —

But if at any instant of time, whilst the wind was blowing it was observed, that when the water stood at E, the top of the tube out of which it is forced, it was depressed in the other tube to some given level BF, the altitude at which it would have stood in each, had it immediately subsided, may be found in the following manner:—

Let b = AB or EF. Then it is evident, that the column DB is equal to the difference of the columns EF, GF. But the difference of these columns is as $d^2b - d^2x$. Therefore $c^2x = d^2b - d^2x$; and consequently $x = \frac{d^2b}{c^2+d^2}$

For the cases when the wind blows in at the narrow leg of the instrument.

Let $AB - EF = b$, EG or $AD = a$, $GF = DB = x$, and the diameters EH, CA, respectively $= d, c$, as before. Then it is evident, That the column AD is to the column GF as $ac^2 : d^2x$. But these columns are equal. Therefore $d^2x = ac^2$ and consequently $x = \frac{ac^2}{d^2}$ This answers to case I.

It is also evident that the column AD is equal to the difference of the columns AB, DB. But the difference of these columns is as $bc^2 - c^2x$. Therefore $d^2x = bc^2 - c^2x$. Whence we get $x = \frac{bc^2}{d^2 + c^2}$. This corresponds to case II.

As there is always a calculation to be made for every exp. when the legs of the instrument are of unequal bores I would recommend it to the makers of these instruments to make use of tubes that are equal, or at least nearly so, that the error may become next to nothing, it being a thing very easy to be done. In this manner we can readily determine the greatest force, which the wind has blown with, during the time the instrument has been exposed to its action. But as it may be safely left alone, by screwing its spindle into the proper stand, or into the top of a post, as the wind never fails to turn the mouth of it towards itself, it is not necessary for the Observer to continue always by it, for it may be allowed to stand all nights, exposed to the wind, without any inconvenience, though it should even happen to rain very heavily. However, recourse can only be had to this method of using the instrument on shore: for at sea it must

always be held up in a perpendicular position in the hand, whether it be used when only half full of water or when quite full; which last will be frequently found to be the only practicable method of ascertaining the force of the wind during the night, when it blows so hard that it is impossible to keep any lights on deck. A person filling the wind-gage, in a calm place with water, in order to determine the force of the wind, in the way which I have been just now describing, will be apt to imagine, that it cannot give the measurement correct: for he will find such a repulsion to arise from the edges of the hole G, as to sustain a column of water in the kneed bent tube, perhaps half an inch above the level; but by either blowing across the round hole, or moving his finger over it, he will soon bring the water in the kneed tube to stand at the same level with it; by taking off gradually the convex surface of the water, which projects out at the hole in the form of a drop or spherule. And this effect the wind very soon produces itself. There ought always to be a cover on the top of the tube out of which the water is expelled by the wind; but it should be made very thin. For if there be any such cover, & the mouth of the kneed tube be stopped, after the instrument is quite full of water, in order to prevent the wind from having any influence in raising it, you will find, upon exposing it to all strong gales, that in a very short time it will blow out perhaps half an inch of water. Whence it appears, that a very considerable

error would arise from using the wind gage in this state. But in all the experiments which I have made with this instrument, whilst it had the cover and the round hole of $\frac{2}{10}$ ths of an inch in diam. in the middle of it. I have not been able to discover any error. The use of the small tube of communication a b. (Fig. 1) is to check the undulation of the water, so that the height of it may be read off from the scale with ease & certainty. But it is particularly designed, to prevent the water from being thrown up to a much greater or less altitude, than the true height of the column which the wind is able at that time to sustain, from its receiving a sudden impulse, whilst it is vibrating either in its ascent or descent. For water in the legs of a Siphon is capable of being put into a vibrating motion like a pendulum; [u] and therefore if acted upon when in ascent, the height which it ascends to will come out greater than the truth & less if acted on in the descent. -

The height of the column of water sustained in the wind gage being given, the force of the wind upon a foot square is easily had by the following table, and consequently on any known surface.

[u] Newtoni Princip. Mathemat. lib. II prop. XLIV. theor. XXXV.

Table I.

Number	Height of the water in the wind-gage	Force of the wind on the foot square in avoirdupois pounds	Common designation of such a wind
1	12 Inches	62.5	
2	11	57.293	
3	10	52.083	most violent hurricane
4	9	46.875	
5	8	41.667	very great d°.
6	7	36.548	great hurricane
7	6	31.75	hurricane
8	5	26.041	very great Storm
9	4	20.833	great d°.
10	3	15.625	Storm
11	2	10.416	very high wind
12	1	5.208	high wind
13	$0\frac{1}{2}$	2.604	brisk gale
14	$0\frac{1}{10}$	.521	fresh breeze
15	$0\frac{1}{20}$	.260	pleasant wind
16	$1\frac{1}{40}$	.130	gentle wind

Example

If it were required to know the force of the wind, when the column of water sustained was equal to $4\frac{6}{10}$ inches. Then by Tab I

	pounds
4 Inches. =	20.833
0.5 or $\frac{1}{2}$ inch =	2.604
0.1 =	0.521
Sum 4.6 =	23.958 = force on every Square foot

Any change that can happen in the Specific gravity of the Water from heat or cold will make no sensible alteration on experiments made with this instrument. –

A cubic foot of water is generally supposed to weigh 1000 ounces Avoirdupois; and from some experiments made by M^r. Mus-schenbroek it would appear, that betwixt freezing & boiling or in 180° on Fahrenheits scale, it increased only $\frac{1}{85}$ = .0117 of its whole bulk or volume (b). I cannot however find any Author

(b) Musschen. Introduc. ad Philos. Natur. tom. II p. 625.

that mentions at what precise degree of heat a cubic foot of water was weighed. Mr. Fahrenheit indeed made several of his curious experiments on the specific gravities of bodies when the water raised ~~this~~ thermometer to 48°.(c) Now if we suppose the greatest heat of the water which we make use of in the wind-gage to be 90°, which exceeds 48° by 42, the greatest change produced will be only .0027 or $\frac{27}{10000}$ parts of the whole. So that if the altitude of the column of water sustained by the wind were even to be five inches, the part of this effect, arising from the diminution of the specific gravity of the water occasioned by the greatest heat, will only amount to 0.0135 or $\frac{135}{10000}$ parts of an inch, a change which cannot be measured by the instrument. It may be sometimes necessary to employ other fluids besides water, particularly if the degree of cold be below freezing; for then we must use a fluid that will not freeze in the degree of cold in which we expose the instrument, otherwise the wind can have no influence on it, and the liquor freezing in the tube will break it. I shall therefore, mention a few liquors in the following table that will answer the purpose, as also a general method of reducing them all to one common measure. But of all the fluids I am acquainted with, when the effects of frost are to be feared, I know none better adapted to our purpose than a saturated solution of sea-salt, since it does not freeze till the thermometer falls to 0 degrees, and is a fluid constantly of the same specific gravity. Spirit of wine, independent of its being more variable in respect of specific gravity by the influence of heat & cold, is also more or less so, as it is more or less rectified. And although the true specific gravity were known at the beginning of the operation

(c) Philosophical Transactions No. 383.

it would even change during the time of using it, by imbibing Moisture from the Air.

Let w represent the weight of a column of water, having its altitude measured by one of the divisions on the Scale, and its base to any given surface whatever; and let n denote in general the number of these divisions which measures the whole length of the column of the water which the wind sustains. Then nw will represent always its weight, and will serve as a common Multiplier for the Specific gravities of all other liquors.

Table II. —

Names of liquors.	Specific gravities	Common multiplier	weight measuring the force of the wind.
Water. —	1.0000		nw
Sat. Sol. of Salt.	1.244.	$nw.$	$1.244 \times nw$
Urine	1.030.		$1.030 \times nw$
Ditto - - - -	1.016		$1.016 \times nw$
Alkohol. - - - -	0.825		$0.825 \times nw$
Proof Spirits	— 0.927		$0.927 \times nw.$
	&c. &c.		&c. &c. — —

Example

Let w represent the weight of a Column of water $\frac{1}{20}$ of an inch high, standing on a square foot; and let $n = 80 = 4$ inches. Then by (Tab I) $nw = 20.833$ Avoird. ~~pounds~~ Therefore 1.244×20.833 = weight of a saturated solution of sea salt of the same altitude. and $\frac{4}{1.244}$ = the altitude of a column of a saturated solution of ~~sea salt of the same altitude~~ the same, weighing 20.833 pound Avoirdupois. w may represent a square yard, the surface of a Sail &c. —

If the velocity & density of the wind in any particular case were accurately determined, this instrument which gives its force or momentum, would enable us to ascertain the velocity in every other case. the density being known. For it appears from experiments made by Mr. James Ferguson F.R.S. on the whirling table that its force is as the square of the velocity. But as the density. which is one of the data requisite for determining the velocity by this instrument was not taken into consideration in these experiments, all we can do at present is to suggest the Idea. -

It may not, perhaps, be improper to take notice, that evaporation will have some effect in diminishing the altitude of the columns of water; though its influence, for the most part will be very inconsiderable. The more frequently, therefore, the instrument is examined, it will be so much the better. If it be exposed to the action of the wind, whilst it happens to snow, it will be necessary to look at it frequently, least the snow should choak up the mouth of the wind-gage. - -

Extract of a letter from Dr. Lind to Col. Roy. Dated Edin. May 26, 1775 - -

The wind-gage ought to be somewhat longer than that I lately sent Sir John Pringle. For we had a gale here on the 9th current, which supported a column of water of $6\frac{7}{10}$ inches, whereas that I sent was not so long

The force of this gale on a Square foot was equal to 34.921 libs Avoirdup., and it has done great damage to our gardens. West India hurricanes would require gages of a Still greater length to measure them. —

Finis —

Fig 2

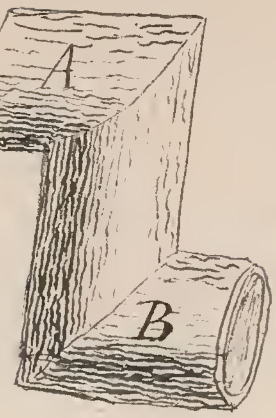
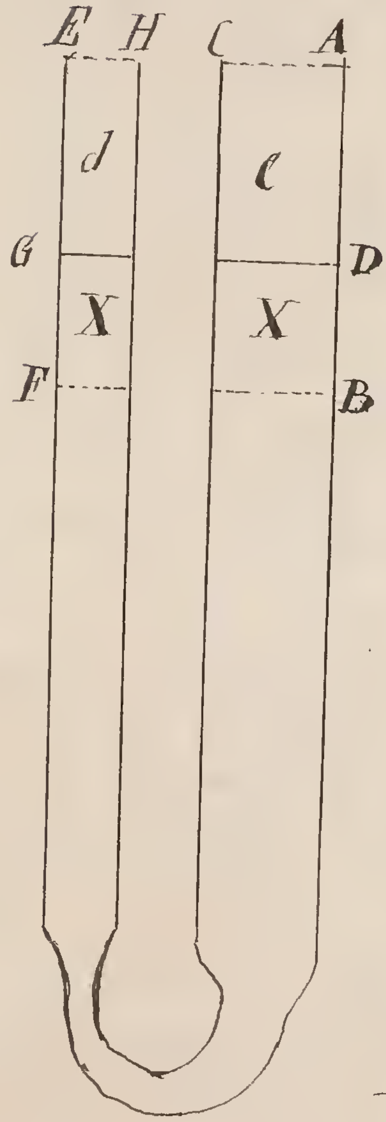


Fig 3



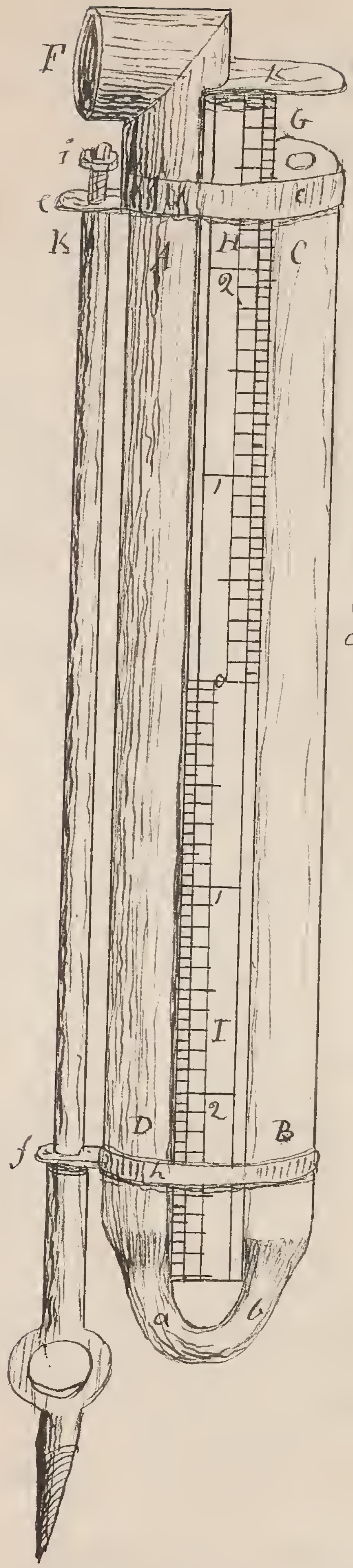


Fig 1.

An Investigation into the principals of Pro-gressive & Rotary Motion

The communication of motion from impact is well known to constitute a considerable part of that branch of naturall Philosophy called Mechanics & as all our inquiries therein are directed, either to assist us in those operations which add to the conveniencies of life, or to explain for the satisfaction of the mind, those changes which we daily see arise from the effects of bodies on each other, it might naturally have been expected that the attention of Philosophers would have been engaged, first in the investigation of such cases as most frequently occur from the accidental action of one body upon another, before they had proceeded to others less obvious. A little consideration will convince any one how seldom it happens, in the collision of two bodies, that their Centers of gravity & point of Contact lye in the line of direction of the striking body, yet few writers on Mechanics have extended their enquiries any further than this simple case. It must however be acknowledged, that the actions of bodies on each other, in directions not passing throw their centers of gravity, affords a subject at least curious in speculation; for my own part, I have little doubt but that it might be rendered extremely usefull to the practical Mechanic. J. Bernoulli was the first who published any thing on this subject. He found the point about which a body at rest would begin to revolve when struck by another

body, observing however that D. Bernoulli had also discovered the same: he has also observed the curve described by that point in the progressive motion of the body, & has directed a method of enquiry by which the velocities of the bodies may be found after the stroke which comprehends all he has done on the subject. Two years afterwards D. Bernoulli published a paper on progressive & rotatory motion, containing nothing more than what J. Bernoulli had before given us & what is a little extraordinary says in his introduction, de tali quidem percussione nihil adhuc, quantum scio, publici juris factum fuit ab iis qui de motu corporum a percussione egerunt. Euler has also investigated the velocities of the bodies after impact in a manner somewhat different, but has rendered it much more intricate by a fluxional calculus. To any one, however, who attentively considers the subject, the theory must still appear to be extremely imperfect, as, independent of principals not more self evident than the propositions they are intended to demonstrate, which both J. & D. Bernoulli have assumed in their investigations, a great variety of other circumstances equally interesting naturally arise in an inquiry into this matter, circumstances absolutely necessary towards understanding the principles of the motion of bodies after impact. This induced me to consider the subject with some attention, & presuming that I have not been altogether unsuccessfull in my endeavours to render the theory more perfect, I have determined to lay the result of my enquiries before the Royal Society. I thought it considered for the sake of perspicuity

to divide the whole into distinct propositions; and as the most simple cases are best understood, I have first considered the the case of the Action of a Body on a lever having a corpuscle at each end; & I was the more induced to treat the subject in this manner, as most of the principals can be immediately applyed to any number of corpuscles, in consequence of which the general investigations are rendered more easy & satisfactory. —

Prop. I.

Let A & B be two indefinitely small bodies connected by a Lever void of Gravity, and suppose a force to act at any point D perpendicularly to the lever, to find the point about which the bodies will begin to revolve. —

From the property of the lever, the effect of the force acting at D (fig 1.) on the body A is to the effect on B as BD : AD; hence the ratio of the spaces Am, Bn described by the bodies A & B in the first instant of their motion, will be as $\frac{BD}{A} : \frac{AD}{B}$ join mn, & if necessary produce that line & AB to meet in C, which will manifestly be the point about which the bodies begin to revolve. Hence from similar figures $BC : AC :: \frac{AD}{B} (\propto Bn) : \frac{BD}{A} (\propto Am) :: A \times AD : : B \times BD$, or $DC - DB : AD + DC :: A \times AD : B \times BD$, and consequently $DC = \frac{A \times AD^2 + B \times BD^2}{B \times BD - A \times AD}$ and therefore D is the Center of percussion or oscillation to the point of suspension C

Cor 1. Hence, whatever be the magnitude of the stroke at D, the point C will remain the same. —

Cor 2. If the force acts at the Center of gravity G, the bodies

will have no circular motion for in this case $B \times BD - A \times AD = 0$, & therefore DC becomes infinite

Cor 3^d. If the force acts at one of the bodies, the center of rotation C will coincide with the other body ~

Cor. 4. If the lever had been in motion before the stroke, the point C, at the instant of the stroke, would not have been disturbed

PROP. II

Let a given quantity of motion be communicated to the Lever at D, to determine the velocity of the Center of Gravity G.

The Space Am, described by the body A in the first instant of motion, is as $\frac{DB}{A}$: now $CG = CD - DG = CD - AG + AD = \frac{A \times AD^2 + B \times BD^2}{B \times BD - A \times AD} - AG + AD = \frac{B \times BD \times BG + A \times AD \times AG}{B \times BD - A \times AD}$; also $CA = CD + DA = \frac{A \times AD^2 + B \times BD^2}{B \times BD - A \times AD} + DA = \frac{B \times BD \times AB}{B \times BD - A \times AD}$: hence we have $\frac{B \times BD \times AB}{B \times BD - A \times AD} (AC) : \frac{BD}{A} (mA) :: \frac{B \times BD \times GB + A \times AD \times AG}{B \times BD - A \times AD} (CG) : \frac{B \times BD \times GB + A \times AD \times AG}{A \times B \times AB}$ & Gw the velocity of the Center of gravity: hence if the motion be communicated at G, the velocity becomes as $\frac{B \times GB^2 + A \times AG^2}{A \times B \times AB}$.

Let now the motion, which is supposed to be actually communicated to the rod at D, be equivalent to the motion of a body whose magnitude is G, and moving with a velocity v; then if that motion be communicated at G, the velocity of the center of gravity is well known to be $= \frac{G \times v}{A+B}$; hence $\frac{B \times BG^2 + A \times AG^2}{A \times B \times AB} : \frac{B \times BD \times BG + A \times AD \times AG}{A \times B \times AB} :: \frac{G \times v}{A+B} : \frac{G \times v}{A+B} \times \frac{B \times BG \times BD + A \times AD \times AG}{B \times BG^2 + A \times AG^2}$ = velocity of the

center of gravity, when the same motion is actually communicated to any point D. Now $BD = BG + GD$ & $AD = AG - GD$; hence $B \times BG \times BD + A \times AD \times AG = B \times BG^2 + A \times AG^2 + GD \times \overline{B \times BG - A \times AG} =$ (because $B \times BG - A \times AG = 0$) $B \times BG^2 + A \times AG^2$; consequently the velocity becomes $\frac{G \times V}{A+B}$; & hence the center of gravity moves with the same velocity, wherever the motion is communicated.

Prop. III.

Let a given elastic body P, moving with a given velocity, be supposed to strike the lever at the point D in a direction perpendicular to it; to determine the velocity of the center of gravity G after the stroke.--

Suppose first the body to be nonelastic, & let v be the velocity of the center of gravity after the stroke on that supposition, and V the velocity of the striking body: then $CG : CD :: v : \frac{v \times CD}{CG}$ = the velocity of the point D after the stroke, or of the body P; for the same reason $\frac{v \times CA}{CG}$ and $\frac{v \times CB}{CG}$ equal the velocities of A & B respectively. Now because in revolving bodies the momenta, arising from the magnitude of the bodies, their distance from the center of rotation and velocity conjointly, remain the same after the stroke as before, we shall have $P \times V \times DC = \frac{v \times CD^2 \times P}{CG} + \frac{v \times CA^2 \times A}{CG} + \frac{v \times CB^2 \times B}{CG}$, and therefore $v = \frac{P \times V \times DC \times CG}{P \times DC^2 + A \times AC^2 + B \times BC^2} = \frac{P \times V \times CG}{\overline{A+B} \times CG + P \times DC}$ for the velocity of the center of gravity after the stroke in ipso motus initio.--

Prop. IV.--

To determine the motion of bodies after the first instant, or when they are left to move freely by themselves.--

The writers on mechanics, from considering the equality of motion, on each side the center of gravity, when a body

revolves about that point, have inferred that if a body had a projectile as well as a circular motion communicated to it, the center of gravity would continues to move in a right line, as that point would not be disturbed by the rotary motion; yet, as, in the case we are now considering the bodies begin to revolve about a different center, it may be proper to examine more accuratly into this matter, & to show from what principle it is that the motion of the center of gravity is preserved in a right line.

Let a motion perpendicular to the rod be communicated to A (fig. 2) & then by Cor. 3 prop 1. B will not be disturbed by such an action; and A will in the first instant have a tendency to revolve about B as a center, and would actually describe the arc AH, if the body B were fixed: let the angle ABH be supposed infinitly small, and let GK be the arc, the center of gravity would have described, and draw the tangents AF, Gg to the arcs AH, Gg respectively. Now if A could have moved freely, it would (because AF = AH) have described AF in the same time the arc AH was described, upon supposition that that B was fixed: for the radius AB being perpendicular to the circular arc AH, the force of the lever could have no efficacy to accelerate or retard the motion of A in the arc AH, and therefore the velocity in that arc is the same as it would have been if it had moved freely in the tangent: hence HF is that space through which the centrifugal force of A would have carried that body, could it have moved freely; but as A is connected to B by means of the lever, it is manifest that the same force which would have carried A from H to F in the direction of the lever, will when it has both bodies to move carry it over a space which is to FH as A to A+B or as

Bg : BH, or as gk : FH: hence that Space, or the Space through which the centrifugal force of A will draw the lever in the direction BH, is equal to kg; that is the point k, which is the center of gravity of A & B, will be found at g, and consequently the center of gravity has preserved its Motion uniform in the right line Gg, inasmuch as the centrifugal force, acting perpendicularly to the direction of the center of gravity can neither accelerate or retard its Motion. In the same manner it may be proved, that the Motion of the center of gravity is continued uniform in the same right line whatever be the position of the lever. Moreover, as the centrifugal force acts in the direction of the lever, it cannot alter its angular velocity, which will therefore remain as, in ipso Motûs initio. If now we suppose that to the force impressed upon A, two other equal accelerative forces be communicated to A & B at the same time, it is evident, that no alteration can arise, from the actions of the bodies on each other; & the case will then be similar to the Motion of the bodies, supposing a single force had been impressed at any point D. The like method of reasoning may be extended to any number of bodies. —

The same thing may also be demonstrated in the following manner: The centrifugal forces of A and B (Fig. I.) are respectively A×AC and B×BC; also the centrifugal force of the point G, considering it as having both bodies to move in the direction of the rod, is $\overline{A+B}$×GC, but from Mechanics A×AC+B×BC = $\overline{A+B}$×CG: hence the centrifugal forces of the bodies A & B give the center of gravity a centrifugal force equivalent to its own centrifugal force, which, as the latter would cause that center to move in the tangent Gg, the lever not being fixed at C, it is manifest that the former will

cause the center of gravity to continue its motion in the same direction.

That this motion of the lever, in a direction from the center C, is the only motion which is communicated to it from the effects of the bodies A & B is manifest from hence. The bodies begin to revolve freely about the point C, and consequently if the point C had been fixed, the bodies would have moved on with a uniform angular velocity about C: if therefore we suppose the lever not to be fixed at C, as the efficacy of the centrifugal force which acts in the direction of the lever is now suffered to take place, and no new external force is impressed on either of the bodies, it is manifest, that if in the former case the bodies had no efficacy to disturb the angular velocity of the lever, they cannot have any in the latter, consequently the angular velocity, and from what has been before proved, the uniform motion of the center of gravity in a right line, remain unaltered, after the commencement of the motion.

Prop. V.

In the time bodies make on revolution, the center of gravity will move over a space equal to the circumference of a circle whose radius is CG (fig. I.)

From the last proposition, the angular velocity of the lever is continued uniform; hence the time of a revolution is just the same as if the point C were fixed, and the bodies were to continue to revolve about that point as a center, in which case the center of Gravity G, in the time of a revolution, would evidently describe the circumference of a circle whose radius

is CG. This therefore is the Space the center of gravity describes in a right line when the bodies move freely, for from the last prop. that center is carried uniformly forward with the same velocity.

Cor. 1 Hence if the magnitude of the force acting at D vary, the velocity of the center of gravity will vary in the same ratio as the angular velocity.

Cor. 2. Hence the point D may be found, where a force being applied, the bodies shall make one revolution, whilst the center of gravity moves over any given space (S): for let p = periphery of a circle whose radius is unity then $p:1::S:\frac{S}{p}$ = the radius of a circle whose circumference is the Space to be passed over in the time of a revolution, and which must therefore, by Proposition, be equal to GC; the point C therefore being determined, D may easily be found, for from Mechanics $CG \times DG$ is given; & from Cor. 1. Prop. 1. When D comes to A, C will coincide with B; $CG \times DG = AG \times GB$, and consequently $DG = \frac{AG \times GB}{CG}$

Prop. VI.

To determine the time of a revolution, supposing every thing given as in Prop. 3.

The point D being given, we have from Cor. 2. to the last Prop. $CG = \frac{AG \times GB}{DG}$; put w = the circumference of a circle whose radius is CG, and it appears from the last Prop. that w is the Space the center of gravity passes over in one revolution; hence because from Prop. 4 the center of gravity moves uniformly, we have by Prop. 3 $\frac{2 \times V \times P \times CG}{\overline{A+B} \times CG + P \times DC} : 1'' :: w : w \times \frac{2 \times V \times P \times CG}{\overline{A+B} \times CG + P \times DC}$ = the time of one revolution.

Cor. Hence the angular velocity being inversely as the time of a revolution, will vary as $\frac{\overline{A+B}\times CG+P\times DC}{V\times P\times CG\times W}$.

Prop. VII.

The point G, as the center of gravity moves forward, will describe the common cycloid. -

From the description of the common cycloid it appears, that the center of the generating circle passes over a space equal to the circumference of that circle whilst it makes one revolution. With the center G (fig. 3.) and radius GC, describe the circle cxy, and draw CR, GW, perpen. to ABC, & let the circle cxy revolve on the line CR; then will the center G move over a space equal to the circumference of a circle cxy whilst it makes one revolution, and the point c will describe the common cycloid: but from Prop. 5. the point G will move over a space equal to the circumference of a circle whose radius is GC, whilst the bodies, & consequently GC, make one revolution; & hence the point C will describe the same curve as before, that is the common cycloid..

Prop. VIII. -

Let a motion be communicated to the lever obliquely, to determine the point about which the bodies begin to revolve.

Let D (fig. 4.) represent the force communicating the motion at the point D, which will resolve into two others FH, HD, the former FH parallel to the lever, & the latter HD perpendicular to it. Let C be the point about which the bodies would have begun to revolve, had the force HD only acted, and which may be found by Prop. 1: & suppose in this case mgn to have been the next positions of the lever after the commencement of

the motion, or that the bodies A, B, & center of gravity G, had been carried to m, g & n respectively. But as the force FH acts at the point D at the same time in the direction of the rod: if we take Gq : Gg :: FH : HD, then whilst the center of gravity would have moved from G to g in consequence of the force HD it will by means of the force FH be carried in the direction of the lever from G to q, & also every other point of the lever will be carried in the same direction with the same velocity; take therefore Ap & Br each equal to Gq, & compleate the parallelograms Aa, Gw & Bb, and the bodies A, B, & center of gravity G will, at the end of that time, be found at a, b & w respectively, and awb will be the position of the lever. Now it is evident, that C is not the point about which the bodies begin to revolve, for (considering the lever to be produced to C) that point must have moved over a space Cc = Gq, when the lever is come into the position awb; draw CO perpen. to CB, & GO perpen. to Gw, & O will be the center of rotation at the commencement of the motion. For conceive CO to be a lever, then the lever ABC has a circular motion about C, whilst that point is moved from C to c, & consequently the point O is carried forward in a direction parallel to Cc by this motion; but as the lever CO is carried by a circular motion about C in a contrary direction, it is evident that that point of the lever CO must be at rest where these two motions are equal, as they are in contrary directions. Now the velocity of C in the direction Cc : velocity of G about C :: Gq : Gg :: (by sim. trian) CO : CG, and the velocity of the point G about C : velocity of

the point O about C :: CG : CC; hence ex æquo the velocity of C in the direction Cc, or of O in the direction OP parallel to Cc, is equal to the velocity of the same point O in a contrary ~~parallel~~ direction arising from its rotation about C, & consequently O being a point at rest must be the center of rotation in ipso motus initio. Also because m a is equal & parallel to nb, ab must be equal & parallel to mn, therefore the angular velocity is just the same as if the force FH had not acted. The center O of rotation at the beginning of the motion being thus determined, every thing relative to the motion of the bodies, after they are at liberty to move freely, may be determined as in the preceding propositions. –

Cor. 1. Hence it appears, that whatever be the magnitude or direction of the force communicating the motion, or the point at which it acts, the center of gravity will move in a line parallel to the direction of the force, for the triangles FHD, Gqw being similar, Gw must be parallel to FD. –

Cor. 2. The same is manifestly true for any number of bodies; for let (fig. 5) E be a third body, & conceive it to be connected with the other two bodies A & B in their center of gravity G; then if FD represents the force acting at the point D, it is evident from the last Corol. & the second Prop. that the center of gravity moves with the same velocity & in the same direction, as if the same motion had been communicated at G in a line RG parallel to FD, & that the

center of gravity has the same velocity communicated to it, as if the two bodies had been placed at G; conceive therefore the bodies A & B to be placed at G, & let the force act at D & then from the last Corol. the center of gravity g, of the three bodies, will move in a line parallel to the direction of the force communicated. In the same manner it may be proved for any number of bodies. —

Scholium.

The method here made use of to determine the point of rotation *in ipso motus initio*, when a single force acts at any point D, may be applied, when any number of forces act at different points at the same time for let (fig. 1.) α, β, γ, &c. represent the forces acting on the lever at the points D, E, F, &c. respectively; then from the same principles the effect of all the forces on A : effect on B : : $\frac{\alpha}{AD}+\frac{\beta}{AE}+\frac{\gamma}{AF}$ &c : $\frac{\alpha}{BD}+\frac{\beta}{BE}+\frac{\gamma}{BF}$ + &c which quantities put equal to P & Q respectively, & then $\frac{P}{A} : \frac{Q}{B} :: Am : Bn :: AC : BC$, from whence it appears, that (putting $GC+GA=AC$ & $GC-GB=BC$) the distance $GC=\frac{A\times Q\times AG+B\times P\times BG}{B\times P-A\times Q}$. The same conclusion might have been deduced from this consideration; that if any number of forces act on a lever, the effect on any point of that lever is just the same as if a force, equivalent to the sum of those forces, had acted at their common center of gravity, find therefore their common center of gravity, & conceive a force equivalent to them all to be communicated to that point & the problem is reduced to the case of the first Proposition. If any of the forces

had acted on the opposite side of the lever, such force must have been considered as negative. —

If there be any number of bodies placed on the lever, & a single force acts at D, it will appear from the same principles that the point C, about which they begin to revolve, will be the point of suspension to the center of percussion D; & the same conclusion will be obtained, if the bodies be not situated in a straight line. As a direct investigation, however, is always to be preferred to conclusions drawn from induction, it may be thought proper before we apply any of the foregoing principles to the case of the actions of bodies on each other by impact, to shew how such a direct investigation to determine the point about which a body, having a motion communicated to it, begins to revolve, may be obtained: previous to which, however some further considerations are necessary. — —

PROP. IX. —

If a force acts upon a body in any given direction not passing throw the center of gravity; to determine the plane of rotation, the direction in which the center of gravity begins to move, and its motion after. —

Conceive a plane ABYZ (fig. 6) to be supported upon a line AB passing through its center of gravity G, and suppose a force to act at any point D in that line, & in a direction perpin. to the plane; then it is manifest, that such a force can give the plane no rotatory motion about AB. Imagine now the support to be taken away whilst the force is acting at D, then it is evident, that as the plane had no tendency to move about AB as an axis, and the taking away of the support can give it no such motion, it will, by Cor. 2

Prop. 3. begin its progressive motion in the direction in which the force acts; and as the force is supposed not to act at the center of gravity, it must at the same time have a rotatory motion about some axis, which, as it has no motion about AB, must lie some where in the plane, perpendicular to AB; & consequently in ipso motus initio the plane of rotation must be perpen. to the plane AyBZ. Let LCM, perpendicular to AB, be the axis about which the plane begins to revolve, and p, q be two equal particles in the plane similarly situated in respect to AB, also qb, pa perp. to LCM. Now the centrifugal force of p, or its force in the direction ap is p×ap, & that of q in the direction bq is q×bq; to determine now how these forces will affect the motion of the plane, we may observe in the first place, that the force p×ap acting at a in the plane, must tend to give it a motion about an axis perpend. to the plane; but as an equal force q×bq acts at q to give it a motion in a contrary direction, it is evident that the two forces will destroy each other, so far as they tend to generate any motion in the plane about an axis perpend. to it, & hence it is manifest, that if the parts of the plane AyB, AZB, be similar, & similarly situated in respect to AB, the plane, after the commencement of the motion, will have no tendency to revolve about an axis perpend. to it. Also, as the centrifugal force of each particle acts in a direction parallel to AB, it can give the plane no tendency to revolve about that line as an axis, and consequently the plane of rotation will be preserved as in ipso motus initio. Conceiving therefore the plane on each side the line AB to be similar, and similarly situated, suppose another plane to be fixed upon this, whose parts

on each side AB are similar, and similarly situated, and the force to act as before, then it is manifest, that as each plane endeavours to preserve the same plane of rotation, ~~for the action of one plane on another~~ the two planes connected will also continue to move in the same plane of rotation, for the action of one plane on another, on each side of the plane of rotation being equal, cannot tend to disturb the motion in that plane; and as this must be true for any number of planes similar & similarly situated, it is evident, that if a force should act on a body, and each section, perpend. to the direction of the force, should be similar on each side the plane passing through the direction of the force, and the center of gravity of the body, that that plane would be the plane of rotation in which the body would both begin & continue its motion. It appears also from what has been proved, that if every section on each side that plane had not been similar, the plane of rotation would not necessarily have continued the same after the commencement of the motion. Hence all bodies, formed by the revolution of any plane figure, will have the axis about which they were generated, a fixed axis of rotation; to determine, however, every other axis of a body about which it would continue to revolve, would be foreign to the subject of this paper. Supposing therefore the plane of rotation to continue the same (for in this paper I mean to confine my enquiries to such cases) imagine all the particles of the body to be reduced to that plane orthographically, which supposition not affecting the angular motion of the body, the centri-

-fugal force of all the particles, to cause the body to revolve about an axis perpendicular to that plane, will remain unaltered. Let LMNO (fig. 7.) be that plane, & suppose a force to act at A in the direction PA lying in the same plane, which produce untill it meets LN, passing through the center of gravity G, perpendicularly in D; then by Cor. 2. Prop 8. the center of gravity G will begin its motion in a line pa-rallel to PA, or perpend. to LN; & consequently the center C, about which the body begins to revolve, must lie somewhere in the line LN. Now the centrifugal force of any particle p is p×pc; let fall pa perpend. to LN, then the effect of that force, at C, in a direction perpendicular to LN, will be p×pa, and in the direction CL, it will be p×Ca; but as the sum of all the quantities p×pa=0, and the sum of all the quantities p×Ca = the body multiplied into CG, it follows from the same reasoning as in prop. 11. that the point G will continue to move in a direction perpend. to LN; and also, as the forces p×Ca act in a direction perpend. to that in which the center of gravity moves, its motion must be continued uniform. In the following Propositions, therefore, we sup-pose the axis of the body, after the commencement of the motion, to continue perpendicular to the plane passing thro' the direction of the force, & the center of gravity of the body, & that the body itself orthographically projected upon that plane; also in the case of the action of two bodies on each other, the plane passing through direction of the striking body and point of percussion is supposed to pass through

the center of gravity of each body; that the axis of each body after it is struck continues perpendicular to that plane, and that each body is reduced to it in the manner above described. —

Prop. X. —

To determine the point about which a body, when struck begins to revolve

Let LMNO (fig. 7.) represent the body, G the center of gravity, & PA the direction of the force acting at A, which produce till it meets LN, passing through G, perpendicularly in the point D; draw pb perpendicular to pc, on which (produced if necessary) let fall the perpend. Dw; C being supposed the point about which the body begins to revolve, & which, from the last Prop. is somewhere in the line LN. Because the body in consequence of the force acting at D begins to revolve about C, & consequently if immediately after the beginning of the motion a force were applied at D equal to it, & in a contrary direction, the motion of the body would be destroyed, it is evident, that the efficacy of the body revolving about C, to turn the body about D, should any obstacle be opposed to its motion at that point, must be equal to nothing; for were it not, the body, when stoped at D, would still have a rotatory motion about that point, & consequently two equal & opposite forces applied at D would not destroy each others effect, which would be absurd. Now the force of a particle p, in the direction pw, being p × pc its efficacy to turn the body about the point D is p × pc × Dw;

but by Sim. Triang. $Dw : Db :: aC : pC$, ∴ $Dw = \frac{Db \times aC}{pC}$, and consequently the efficacy to turn the body about $D = p \times Db \times aC = p \times Ca \times \overline{DC - Cb} = p \times Ca \times DC - p \times pC^2$; hence the sum of all the $p \times Ca \times DC$ – the sum of all the $p \times PC^2 = 0$ and consequently $CD = \frac{\text{sum of all the } p \times PC^2}{\text{sum of all the } p \times Ca}$ therefore D is the center of percussion, the point of suspension being at C. —

Cor. From this and the preceding Prop. it appears that every thing which was proved in Prop. 5 & 6 & 7, holds here also in the case of the action of one body on another.

PROP. XI. –

Let a body P (fig. 8.) moving with the velocity V, strike the body Q at rest in the point A, and in a direction AD passing through the center of gravity of the striking body; to determine the velocity of each body after the stroke, supposing them to be elastic. –

The Solution of this Prop. depending on the same principles as Prop 3. we shall have, putting v equal the velocity of the center of gravity G after the stroke, on supposition that the bodies were nonelastic (DGC being supposed perpend. to AB, & C the point about which the body Q begins to revolve) $V \times P \times CD = \frac{v \times P \times CD^2}{CG} + \frac{v \times \text{sum of all the } p \times Cp^2}{CG}$ and consequently $v = \frac{V \times P \times CD \times CG}{\text{sum of all the } p \times pC^2 + P \times CD^2}$; but it is well known that the sum of all the $p \times pC^2 = CG \times CD \times Q$ and hence $v = \frac{V \times P \times CG}{Q \times CG + P \times DC}$ and therefore if the bodies be supposed elastic, we have $\frac{2P \times V \times CG}{Q \times CG + P \times DC}$ for the velocity of the center of gravity G after the stroke. Now to determine the

velocity of P, we have have $\frac{P \times V \times CD}{Q \times CG + P \times DC}$ equal its velocity after the Stroke from single impact, and consequently $V - \frac{P \times V \times CD}{Q \times CG + P \times DC} = \frac{Q \times V \times CG}{Q \times GC + P \times DC}$ is the velocity lost by P from simple impact; hence if the bodies be elastic, $\frac{2 \times Q \times V \times CG}{Q \times GC + P \times DC}$ will be the velocity lost by P if elastic, and consequently the velocity of P after the Stroke $= V - \frac{2 \times Q \times V \times CG}{Q \times GC + P \times DC} = \frac{P \times DC - Q \times GC}{Q \times GC + P \times DC} \times V$.

Cor. 1. If the direction AD passes through G, the CG being equal to CD, we have $\frac{2PV}{Q+P}$ = Q's velocity, and $\frac{P-Q}{P+Q} \times V$ = P's velocity, which is well known from the common principles of elastic bodies.

Cor. 2. If $P \times DC = Q \times GC$, or $P : Q :: GC : D$, then will the body P be at rest after the Stroke. —

Cor. 3. If Q were infinitely great, the velocity of P after the Stroke would be $= -V$ as it ought, for P would the Stroke against an immoveable obstacle. —

Cor. 4. Whatever motion Q gains from the action of P, it would lose, if, instead of supposing P to strike Q, Q were to move in an opposite direction, and strike P at rest with the same velocity with which P struck Q; in such case, therefore, the velocity of Q after the Stroke would be $V - \frac{2P \times CG \times V}{Q \times GC + P \times DC} = \frac{\overline{Q - 2P} \times GC + P \times DC}{Q \times GC + P \times DC} \times V$. —

Cor. 5. Hence if P be infinitely great, or Q be supposed to strike an immoveable object, its velocity after the Stroke will be $= \frac{DC - 2CG}{DC} \times V$: hence when $DC = 2GC$, the body Q will have no progressive motion after the Stroke.

but would in such case, if P were immediatly taken away, continue to revolve about a fixed axis. It may also be observed, that when DC is greater than 2GC, or the velocity of Q is positive, that, because it is impossible for Q to continue its progressive motion, it is only to be understood, that if immediatly after the impact the body P were removed, the body Q would then proceed with such a velocity. — —

Cor. 6. Suppose the bodies to be nonelastic, & let M be the magnitude of a body placed at D, which, being acted upon by P, shall have the same velocity generated as was before generated in the point D of the body Q; then by the common rule for nonelastic bodies, the velocity of M after the stroke will be $\frac{P \times V}{P+M}$, and hence $\frac{P \times V}{P+M} = \frac{P \times V \times DC}{Q \times CG + P \times DC}$. — consequently $M = Q \times \frac{GC}{DC}$..

Cor. 7. If a given quantity of motion were communicated to any point of the body Q, the progressive motion of that body after the stroke would be the same. For suppose the magnitude of the body P to be diminished sine limite, and its velocity to be encreased in the same ratio, then, because $\frac{P \times V \times CD}{Q \times CG + P \times DC}$ (which is the velocity of P after the stroke if the bodies be nonelastic) = (because P is infinitly small) $\frac{P \times V \times CD}{Q \times CG}$, the velocity of P after the stroke from simple impact is finite, consequently its motion must be infinitely small, & therefore P must have communicated all its motion to Q: now in this case the velocity of Q $\left(= \frac{P \times V \times CG}{Q \times CG + P \times CD}\right) = \frac{P \times V}{Q}$

which quantity is independent of the place where the force acts; in the same manner it would appear if we had supposed the bodies elastic.-

Prop. XII.

-Supposing every thing as in the last Prop. except that the direction AD does not pass through the center of gravity g of the striking body; to determine the velocity of each body after the stroke

Let AD (fig. 9.) be produced to meet FgO passing through g, the center of gravity of the striking body, perpend. in F, and suppose O to be the point of the body P which is not disturbed by the action of P on Q: now it appears from Cor. 6. Prop. 11. that if both bodies were nonelastic, and a body equal to $Q \times \frac{CG}{CD}$ were placed at D, the velocity of that body, from the action of P, would be equal to the velocity of the point D of the body Q: for the same reason, therefore, it appears, that if, instead of supposing P to strike Q in the direction FA, a body equal to $P \times \frac{GO}{FO}$ were to strike Q at the same point and in the same direction (which direction is supposed to pass through the center of gravity of that body) the effect on Q would be the same; hence if any quantity $\frac{V \times P \times CD}{Q \times GC + P \times DC}$, which from the last Prop. expresses the velocity of the point D after the stroke, on supposition that the bodies are nonelastic, we substitute for P a body equal to $P \times \frac{GO}{FO}$ we shall have $\frac{V \times P \times DC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ for the velocity of the point D from the action of P: and consequently $\frac{2 \times V \times P \times GC \times gO}{Q \times GC \times F + P \times gO \times DC}$ = velocity of the center of gravity G of the

body Q, after the Stroke, if the bodies be perfectly elastic, To determine now the velocity of the striking body, let Of perpend. to Og, be the space described by the point O in the first instant of time after the Stroke, which, as that point is not disturbed by the action of the bodies on each other, may represent the velocity of P before the Stroke, & let Fb represent the velocity of the point F after the Stroke; join fb, and draw gd perpend. to Og, and then will gd represent the velocity of the center of gravity g of the striking body after the Stroke. Draw fc perpend. to FA, and produce gd to meet fc in e; now the velocity lost by P at the point F by simple impact being equal to $V - \frac{V \times P \times DC \times gO}{Q \times GC \times FO + P \times gO \times DC} = \frac{V \times Q \times GC \times FO}{Q \times GC \times FO + P \times gO \times DC}$ we shall have bc the velocity lost by the point F, on supposition that the bodies are perfectly elastic (supposing Of to represent the Value of V) equal to $\frac{2 \times V \times Q \times GC \times FO}{Q \times GC \times FO + P \times gO \times DC}$ & therefore by sim. triang. fc(FO) : cb :: fc(Og) : ed $= \frac{2 \times V \times Q \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ = the velocity lost by the center of gravity g, an hence $V + \frac{2 \times V \times Q \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ $= \frac{V \times Q \times GC \times FO + V \times P \times gO \times DC + 2 \times V \times Q \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ = velocity of P after the Stroke. Now, as it appears from Prop. 11. that the progressive motion of a body, when left to move freely, continues uniform & in the same direction, it follows, that the expressions for the velocities of each body in the first instant after the Stroke, both in this & the preceding Prop. will represent the uniform progressive velocities with which the bodies will continue to move, and consequently the place of each body, at the end of any given time after impact, may easily be determined. —

Cor. 1 If the direction F'A passes through g, then F'O & gO becoming infinite, we shall have $\frac{2\times V\times P\times GC}{Q\times GC+P\times DC}$ for the velocity of Q and $\frac{V\times P\times DC-V\times Q\times GC}{Q\times GC+P\times DC}$ for the velocity of P, agreeable to what was proved in the last Prop. -

Cor. 2. Hence the point about which P begins its rotatory motion may easily be found; for produce (if necessary) fb & OF' to meet a, and a will be the point required; and by sim. triangles. bc $\left(=\frac{2\times V\times Q\times GC\times F'G}{Q\times GC\times F'O+P\times gO\times DC}\right)$: Cf :: fO (=V) : Oa = $\frac{Q\times GC\times F'O+P\times gO\times DC}{2\times Q\times GC}$, and hence F'a = $\frac{P\times gO\times DC-Q\times GC\times OF'}{2\times Q\times GC}$

Cor. 3. If, instead of supposing Q to be at rest, it had been moving forward in a direction parallel to that of the body P, with the velocity v, the motion of each body after the stroke may easily be determined: for considering P as acting on Q with the velocity $V-v$ we shall have by this Prop. (putting $2M=\frac{2P\times GC\times GO}{Q\times GC\times F'O+P\times gO\times DC}$) $\overline{V-v}\times 2M$ = velocity communicated to G, therefore $v+\overline{V-v}\times 2M$ = velocity of Q after the stroke: also $\overline{V-v}\times M\times\frac{CD}{CG}$ = velocity gained by the point D from simple impact, and consequently the velocity of that point after $=v+\overline{V-v}\times M\times\frac{CD}{CG}$, hence $V-v-\overline{V-v}\times M\times\frac{CD}{CG}$ = velocity lost by P at the point F from simple impact, therefore P's velocity after the stroke $=V-\cancel{V}-v-\overline{V-v}\times M\times\frac{CD}{CG}\times\frac{2gO}{FG}$. In the same manner it might have been determined, had Q moved in an opposite direction.

Cor. 4. Hence also we may easily determine the motion of each body after the stroke, supposing Q had not been moving in a direction parallel to the motion of P, by resolving Q's motion into two parts, one parallel to the motion of P, and the other perpend.; and finding by the preceding what would be the effect of the parallel motions, & then compounding Q's motion after the stroke from that consideration, with the motion it had in a direction perpend. thereto before the stroke.

Cor. 5. The point a of the body P will describe (when that body after the stroke has any progressive motion) the common cycloid.

Cor. 6. Hence, therefore, the times of the revolutions of each body may be determined as in Prop. 6.

Cor. 7. If the bodies had any rotatory motion before impact, every thing relative to the motion of the bodies after the stroke might have been determined from the same principles.

Fig. 1

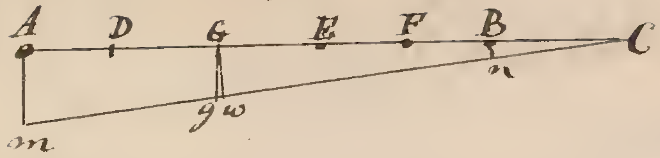


Fig. 2.

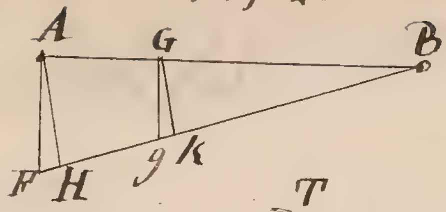


Fig. 3

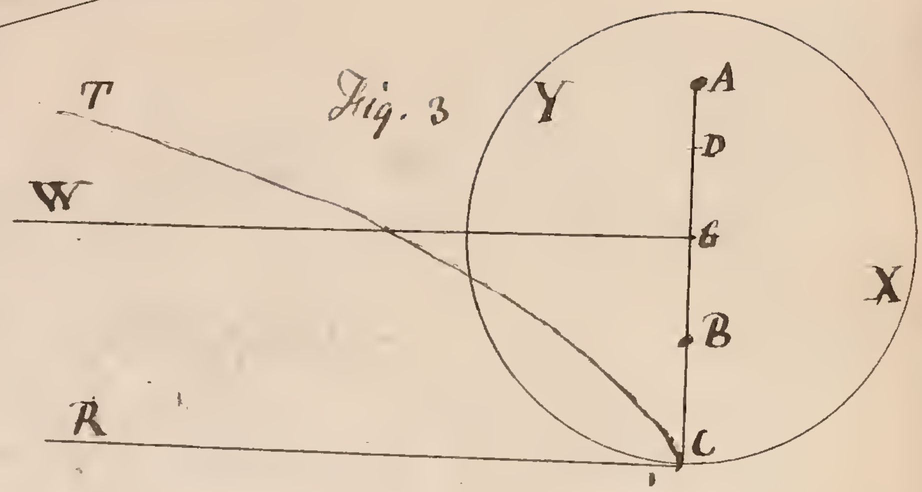


Fig. 4

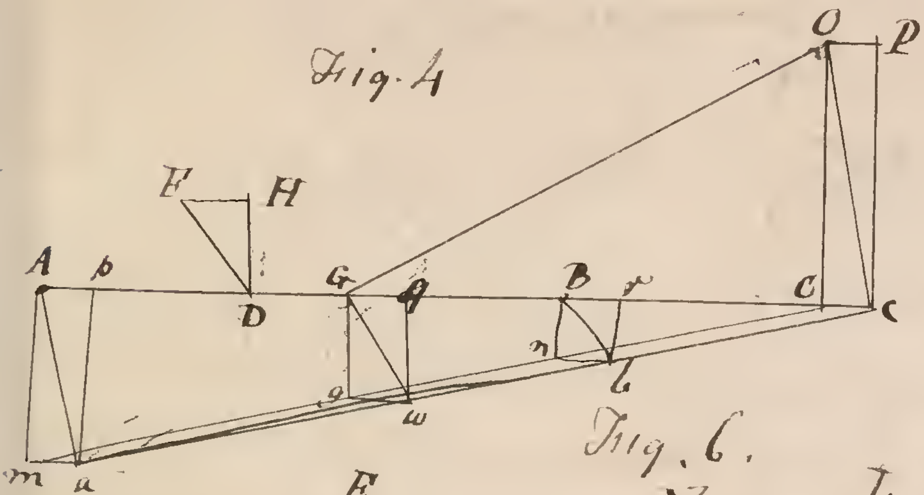


Fig. 5.

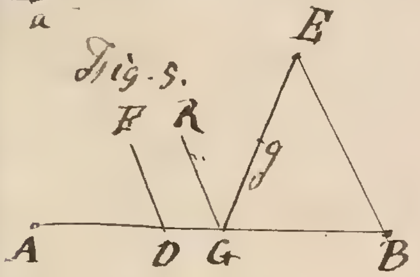


Fig. 6.

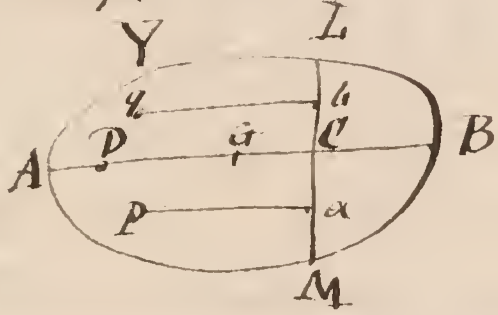
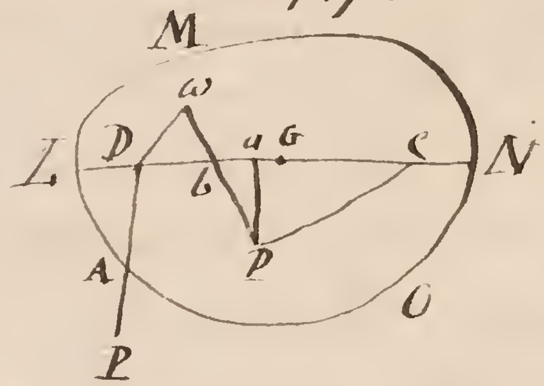


Fig. 7.



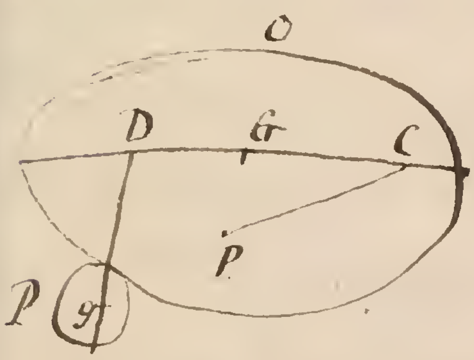


Fig. 8..

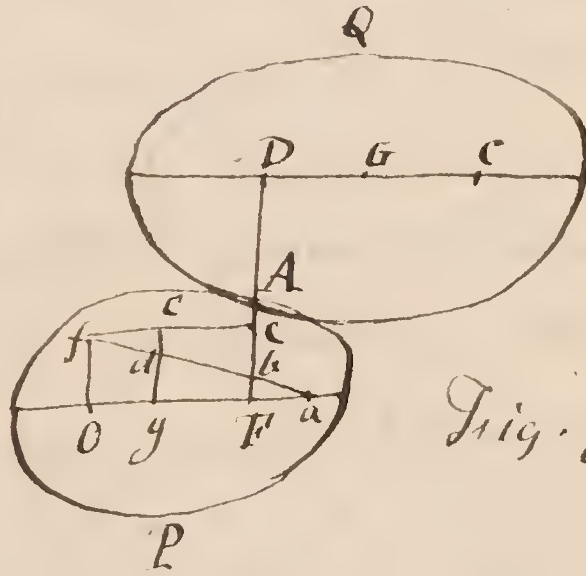


Fig. 9.

Researches upon The Equilibrium of Arches. — *

By M. l'Abbé Bossut. —

The Construction of Arches is well known to be one of the most important objects of Architecture. The Stones destined to form a vault & which of Consequence is called voussoirs ought to be cut in such a manner that they in effect may form by their assemblage the projected vault & that they may support themselves mutually in the air without sliding one on another abstracting from the binding of the mortar or friction. It is in this determination of the proper figure of the voussoirs that consists the art of Trait commonly called Cutting of Stones. But in order that a vault be durable it is not only sufficient that it be constructed on the principals we are going to show; it is necessary it have all the Solidity of which it is capable relative to its figure & the force that acts upon the voussoirs & upon the piers destined to support them.

There are two principal kinds of Vaults the Voutes en berceau & les Voutes en dome We will examine seperately the laws of their Equilibrium —

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* When I read this memorial to the academy in the 1770 · my intention to add some experiments on the strength of Stones; but not having the means of doing it easily & not expecting soon a better I thought proper to publish my Theorie which I have directed to some usefull questions in practice — —

Section first.

Of the Equilibrium of Arches en berceau

1. —

The problem of the equilibrium of arches en berceau has been considered & resolved under different lights by different Geometers. We read in the History of the Academy for the year 1704 that M. Parent considering that the weight of the voussoirs of an arch ought to increase from the keystone to the imposts in order that the forces they exert upon one another & to the adjoining ones mutually destroy he determined of consequence but only by points the figure that the Extrados of an arch ought to have when the intrados was circular; moreover he gave the shoot or push of such an arch against its piers. I do not know if this solution was printed.

Mr. James Bernoulli died in 1705 & left amongst his papers a solution of the problem of the shoot of arches which did not appear untill the 1744 in the collection of the works of that great Geometrician. It is ingenious but imperfect in certain respects; & probably he would have corrected it had he had time to revise it.

In the Memoires of the Academy for the year 1712 M. De la Hire supposed after some experiments that arches whose piers have not a sufficient thickness to support the push broke towards the reins; he of consequence looked upon the upper part of the arch as a wedge which tended to drive away or overturn the piers; & he determined by the Theorie of the wedge & laced the dimensions they ought to have to resist such an effort —

In The Collections of the academy for the years 1729 1730 contains two excellent memorials by M. Couplet upon the push of Arches in bereau. In the first of these memo-rials M. Couplet determines the form & the push of arches & the thickness of their piers in looking upon the voussoirs as infinitely polished: he considered prin-cipally circular arches & he established on the Subject of their equilibrium some Curious & usefull Theorems. In the Second the Author determined the least thickness that we can give to arches uniformly circular; he calculated the push of these Arches in supposing that their voussoirs could not slide upon one another but only seperated when the Arch broke of itself; he gave the thickness of the pier such that the effort composed of the push of the arch & the gravity of the pier is always directed towards some point of the base of the piers from whence results necessarily the equilibrium. —

There has appeared lately some works on the push *1770 of Arches; but all those I know is comprehended in those I came to show: Thus I will dwell no more upon this Historical detail —

Nobody that I know of has considered the matter under the same point of view as those I came to show & I hope that my Researches will be usefull — ——

II.

Fig 1 Let ACB be the intrados of any arch en berceau & acb the extrados. This arch is divided into two similar & equal parts by its vertical axis OCc; & it is composed of the Voussoirs D, X, Y, Y, X', Y' &c which are equal at least in taking two & two on each side D in the middle which forms the keystone. For the first time when the arch comes to be finished & the stone cannot be bound together by the hardning of the mortar. the voussoirs ought to be looked upon as particular bodies under the action of their proper gravity & that of the earth or masonry which they support; the question is then to find the laws of the equilibrium among all the forces which act upon the voussoirs. in such a manner that each of them remain in the place that is assigned to it & that they form by their ~~equilibrium~~ assemblage the figure which the conditions of the problem requires. This Equilibrium being thus established ~~[illegible]~~ will necessarily remain when the parts of the arch will have grown hard or taken band. since being at rest before the Ciment united them their reciprocal & subsequent bonding cannot but keep them in that state. —

II

Supposing that there is applied to each of the voussoirs any forces as V, F, f, F', f' &c directed any way For Voutes en berceau whose piers are of the same height & which are divided into two equal & similar parts by the axis OC the forces acting similarly

on each side of this same axis. Thus the force V which acts upon the key D is vertical; the forces F' & f which act upon the two corresponding Voussoirs X & Y are equal & similarly directed in relation to the axis OC; & so of others. But the solution that we give will apply equally to voultes in arcs rampans & in general to all arches whose those parts OCA, OCB, are not similar & equal, since we go to establish the equilibrium of the adjoining & among all the contiguous Voussoirs. —

IV. —

Let X, X', be two consequential Voussoirs under the respective action of the forces F, F'. The joints m M, n N, p P &c of the Voussoirs ought to be perpend. to the intrados ACB as much for the look of the arch as for the solidity of the construction; & we will suppose them so in effect. Having taken upon the direction of the force F, the part XE to represent it & decompose it into two other forces Xu, Xt, respectively perp. to the two joints m M, n N of the voussoir X. Let X' be the point where the direction of the force Xt meets F'X' of the force F'. I take upon F'X', the part X'E' to represent the force F' & decompose it into two others forces X'q, X'l respectively perpendicular to the two joints n N, p P, of the Voussoir X'. That being supposed it is clear that the two Voussoirs X, X' are in equilibrium if the two forces Xt, X'q, directly opposed, by which they act one against the other are equal. We have only then to form an Equation so that the Force Xt = Force X'q & to substitute for these forces their values. —

V

The parallelogram $XtEu$ is given & the Force Xt = Force $XE \times \frac{\sin.\angle XEt}{\sin.\angle XtE} = F \times \frac{\sin\angle XEt}{\sin.\angle XtE}$ & the parallelogram $X'qE'l$ given the force $X'q$ = Force $X'E' \times \frac{\sin.\angle X'E'q}{\sin\angle X'qE'} = F' \times \frac{\sin\angle X'E'q}{\sin.\angle X'qE'}$.

Thus in the equation Force $Xt = X'q$ will become at first

$$\frac{F\sin\angle XEt}{\sin.\angle XtE} = \frac{F'.\sin.\angle X'E'q}{\sin\angle X'qE'}$$ & it will be

$$(A)\ \frac{F}{F'} = \frac{\sin.\angle XtE \times \sin.\angle X'E'q}{\sin.\angle XEt \times \sin.\angle X'qE'} \ \ldots$$

VI. . . .

Let I be the point of concourse of the Joints mM, nN produced; T that of concourse of the Joint mN, pP, also produced; that the directions of the extreme Joints mM, pP, meet the vertical Axis COG at the points H, L, & that the directions of the forces F, F' meet the same axis at the points Z, G. This being supposed it is clear that the angle $XtE = \angle NIM$ since the sides of the one are perpendicular upon the sides of the other; & for the same reason the angle $X'qE'$ is $= \angle PTN$. Moreover in drawing through the point z where the right line Xu meets the joint mM, the right line zz' parallel to the direction of the force F we will see that the angle XEt, or the $\angle \omega XE$, or the $\angle \omega zz' = \angle uzK - \angle Kzz' = 90^\circ - \angle Kzz' = 90^\circ - (\angle CZF - \angle CHM)$; & by similar considerations the angle $X'E'q = 90^\circ - (\text{ang.} CLP - \angle CGF')$ From whence it will appear $\sin.\angle XEt = \cos.\angle CZF \times \cos.\angle CHM + \sin.\angle CZF \times \sin\angle CHM$ and also the

$\sin. \angle X'E'q = \cos. \angle CGF' \times \cos. \angle CLP + \sin. \angle CGF' \times \sin. \angle CLP.$ the radius being 1. Consequently the equation (A) will be changed into the following. —

$$(B)\ \frac{F}{F'} = \frac{\sin. \angle NIM \times (\cos. \angle CGF' \times \cos. \angle CLP + \sin. \angle CGF' \times \sin. \angle CLP)}{\sin. \angle PTN \times (\cos. \angle CZF \times \cos. \angle CHM + \sin. \angle CZF \times \sin. \angle CHM}$$

VII. —

We see by this Equation that knowing the figure of the intrados, the arcs MN, NP &c to which the Voussoirs is made & the directions of the forces F, F' we will know the relations of the same forces themselves. —

For example if the intrados ACB is a Semicircle; Fig 2 & that each Voussoir is simply under the action of its own gravity; & that the arches of the extrados mn, n'p &c. be concentric & similar to those of the extrados; we can determine by simple elementary geometry the points m, n', p, &c. — —

VIII. — —

Let us return to the general hypothesis of the first figure; & let us suppose that the number of Voussoirs are infinite. Then the arcs MN, NP &c are infiniments petits; & the angles NIM, PTN &c. and those which are formed by the Osculatory Fig. 3 consecutive rays. — or by the radius of Curvature corresponding to those arcs. —

The three Area's MN, NP, PQ being supposed con-secutive, draw to the axis OC the ordinates MR, NR', PR'', QR'''; & from the points M, P, let fall the perp.

Mr, Pd, upon NR', QR''' respectively.

$$\text{Let}\ \left\{\begin{array}{ll} CR \text{ ---------------------------- } & = x \\ CR' \text{ ---------------------------- } & = x' \\ CR'' \text{ ---------------------------- } & = x'' \\ MR \text{ ---------------------------- } & = y \\ NR' \text{ ---------------------------- } & = y' \\ \cancel{P}R'' \text{ ---------------------------- } & = y'' \\ \text{Each of the three areas } MN, NP, PQ, & \\ \text{which I suppose equal between themselves} & = ds \\ \text{The Osculating ray } MI \text{ ------------ } & = R \\ \text{The following Osculating ray } NT \text{ ..... } & = R' \\ \text{the } \angle CZF \text{ that the force } F \text{ makes with axis} & = u \\ \text{The } \angle CGF' \text{ that the force } F' \text{ makes with axis} & = u' \end{array}\right.$$

This being supposed in comparing fig. 1 with fig 3.[2] we will see that the sin. $\angle NIM = \frac{ds}{R}$; sin. $\angle PTN = \frac{ds}{R'}$. Moreover the $\angle CHM = MNr$ we will have cos. $\angle CHM = \frac{rN}{MN} = \frac{dy}{ds}$ & sin. $\angle CHM = \frac{Mr}{MN} = \frac{dx}{ds}$; & in like manner. cos. $\angle CLP = \frac{hQ}{PQ} = \frac{dy''}{ds}$, also sin $\angle CLP = \frac{dx''}{ds}$. Consequently the equation (B) in art. VI, will become $\frac{F}{F'} = \frac{R'}{R} X \left(\frac{dy''.\cos.u' + dx''.\sin.u}{dy \cos.u + dx.\sin.u} \right)$. --

Now $F' = F + dF$; & F being the absolute force which acts upon the area MN, or the resultant of all the forces which act upon each of the points of that area, & which we can consider as equal & parallel; if we call φ any one of these last forces we will have $F = \varphi\, ds$ & $dF = d\varphi\, ds$ or

account of ds being constant. — on the other hand we have $R' = R + dR$, $y'' = y' + dy' = y + 2dy + d^2y$; $dy'' = dy + 2d^2y + d^3y$; $dx'' = dx + 2d^2x + d^3x$; $\cos. u' = \cos. u + d(\cos. u)$; $\sin. u' = \sin. u + d(\sin. u)$. Substituting all these values in the preceeding equation reducing & neglecting the infinitly littles of the third order we will have.

$$2R\varphi.\cos u.d^2y + R\varphi dy.d(\cos. u) + 2R\varphi\sin. u.d^2x + R\varphi dx.d(\sin. u) + \varphi\cos. u.dR\,dy + \varphi\sin. u\,dR\,dx + R\cos. u.d\varphi\,dy + R\sin. u\,d\varphi\,dx = 0$$

We can put this equation under a more convenient form. —

$$(C)\quad \varphi\cos. u.(2Rd^2y + dR\,dy) + \varphi\sin. u.(2Rd^2x + dR\,dx) + Rdy.d(\varphi\cos. u) + Rdx.d(\varphi\sin. u) = 0$$

This equation is the basis of all we can say on this subject. —

IX.

We have now two principal questions to examine. The one consists in finding the figure of the Arch when we know the law of the forces that press the Voussoirs; & the other on the Contrary, to find the law which ought to press the voussoirs when we know the figure of the Arch. We see that the second question is the inverse of the first. The equation (C) will serve us to resolve both: And as it is the business here to make researches applicable to practice without apply-ing myself to useless generalities I will examine

only the cases that have really place in practice at least nearly in nature. I will begin the business by the first of the two Problems. ——

X

Let us suppose that each point of the Curve ACB is pressed vertically throughout with any constant force; then we will have $\varphi =$ a constant quantity & $d\varphi = 0$. Moreover we will have $\sin. u = 0$, $\cos. u = 1$. Consequently the general equation (C) will become here $2R\,ddy + dR\,dy = 0$ where (in multiplying the whole by dy) $2R\,dy\,d^2y + dR\,dy^2 = 0$ whose integral is $R\,dy^2 = A\,ds^2$. -

In order to get an equation between x & y we will put for R its value $-\frac{ds\,dx}{d^2y}$ which will give us $-\frac{ds\,dx.\,dy^2}{d^2y} = A\,ds^2$ or $-\frac{A\,ds.\,d^2y}{dy^2} = dx$, whose integral is $\frac{A\,ds}{dy} = x + C$; from whence we draw $dy = \frac{A\,dx}{\sqrt{\overline{x+C}|^2 - A^2}}$ that is the Equation of the common Catenary. ——

This Equation gives $y = A \times L.\left(\frac{A + x + \sqrt{2Ax + x^2}}{A}\right)$ in compleating the integral so that $x = 0$ then $y = 0$.

With regard to the constant quantity A it determines itself in observing that the given base AB ought to answer also to a given Axis OC. Let us suppose then we make $y = a$ we ought to have $x = b$, a & b being given quantities, we will have to determine A the equation $a = A \times L.\,X\left(\frac{A + b + \sqrt{2Ab + b^2}}{A}\right)$.

We see by this analysis that the surface of the <u>intrados</u>

being pressed vertically by the weights or in general by forces proportional to those weights, all the parts of the arch will remain in equilibrium in giving to it the figure of a Catenary turned upside down which has been known for a long time. —

XI

It happens often that a Vault as for example the arch of a Bridge is loaded with earth or Masonry but to unequal heights above the different Voussoirs. That which forms the key is least loaded the others is pressed more & more in proportion as they are distant on the one side & the other from the middle of the arch. All these different pressures varying in some proportion which ought to regulate the abscissa CR. Suppose then in the second place, that the forces φ being always vertical, as in the preceding case they will be no more constant from one point of the curve to another but in general each force φ will be proportional to some function X of the corresponding Abscissa & will be proportional to the height and figure of the battlement or way that the Vault supports. Then the equation (C) will become (in observing that the Sin. $u=0$, Cos. $u=1$, & puting for φ its value X)

$2RXd^2y + Xdy\,dR + Rdy\,dX = 0$ multiplying the whole by dy we will have

$$2RXdy\,d^2y + Xdy^2dR + Rdy^2dX = 0.$$

Whose integral is $RXdy^2 = Ads^2$. —

Putting for R its value $-\frac{ds\,dx}{d^2y}$ we will have

$$-A\,ds \times \frac{d^2y}{dy^2} = X\,dx$$

whose integral is $\frac{A\,ds}{dy} = \int X\,dx$; which gives for the equation of the curve sought $dy = \frac{A\,dx}{\sqrt{(\int X\,dx)^2 - A^2}}$ —

The constant quantities that the integrations require is determined of themselves always by these considerations that $\frac{dx}{dy} = 0$ when $x = 0$; & that the curve passes through the given points A, C, B. —

XII

If an arch was destined to carry water or even liquid Earth even to a certain point; then the forces φ acting at can be considered as acting perpendicularly to the curve ACB; which is a third case that is a propos to examine.

I suppose then that to each point of the curve ACB, there is a force acting perpendicularly proportional to any fonction X of x & its constantes. —

Then we will have $\sin. \omega = \frac{dx}{ds}$, $\cos. \omega = \frac{dy}{ds}$, $\varphi = X$

Consequently the equation (C) will become

$$X\,dy(2R\,d^2y + dR\,dy) + X\,dx(2R\,d^2x + dR\,dx) + R\,dy.\,d(X\,dy) + R\,dx.\,d(X\,dx) = 0 \text{ —}$$

or $\quad 3RX(dy\,d^2y + dx\,d^2x) + (dx^2 + dy^2) \times (X\,dR + R\,dX) = 0$

Now on account of ds being constant we have $dy\,d^2y + dx\,d^2x = 0$. Consequently we will have simply $X\,dR + R\,dX = 0$ whose integral is $RX = A$ or $X\,dx = -\frac{A\,d^2y}{ds}$

in putting for R its value $-\frac{ds\,dx}{d^2y}$, then $\int X dx = B - \frac{A dy}{ds}$; which gives $dy = \frac{(B - \int X dx) \times dx}{\sqrt{A^2 - (B - \int X dx)^2}}$; the Equation of the curve sought.

For example, suppose that the vault ACB carried Fig 4 a true fluid represented by ACBHZKQ & that VC be the known height of that fluid above the Key Stone; then the perpendicular pressure upon each point of the Area MN will be represented by the vertical line $MT = RC + CV = x + a$ in making $CV = a$; thus $X = x + a$ & $\int X dx = \frac{x^2}{2} + ax$. Consequently the equation of the curve will be $dy = \frac{(2B - x^2 - 2ax) dx}{\sqrt{4A^2 - (2B - x^2 - 2ax)^2}}$ or (in observing that we ought to have $\frac{dx}{dy} = 0$ when $x = 0$)

$$dy = \frac{(2B - x^2 - 2ax) \times dx}{\sqrt{(x^2 + 2ax)^2 - 4B(x^2 + 2ax)}}$$

The finished equation ought to be such the $x = 0$ should give $y = 0$ & that $x = 0$, $C = c$ gives $y = OB = b$, b & c being given quantities

XIII.

We will examine still a fourth case which occurs sometimes in practice, that where each point of the curve is pressed with two forces the one vertical & the other perpendicular to the curve; the forces of the first kind can be considered as arising from a weight the same as the voussoirs; & that of the second the pressure of a fluid that covers the vaults ——

Let MI be the vertical force from the point M, Mh the force perpendicular to the curve; & let the parallelogram $Mlgh$ be compleated. The diagonal Mg will express the force whee have named φ, & the angle gMI will be equal to that which has been named u produce Ml & make gf perpendicular to it & represent the vertical force Ml by p, the perpendicular force Mh by X (p & X being any constant function of x & of its constants). It is clear that we will have $gf = lg \times \sin. glf = Mh \times \sin. MN\theta = \frac{X\,ds}{ds}$; $lf = Mh \times \cos. MN\theta = \frac{X\,dy}{ds}$; $Mf = p + \frac{X\,dy}{ds}$; But on the other hand $gf = Mg \times \sin. gMf = \varphi \sin. u$; $Mf = Mg \times \cos. gMf = \varphi \cos. u$ Thus we will have $\varphi \sin. u = \frac{X\,dx}{ds}$; $\varphi \cos. u = p + \frac{X\,dy}{ds}$; $d(\varphi \sin. u) = \frac{X\,ddx}{ds} + \frac{dX\,dx}{ds}$; $d(\varphi \cos. u = dp + \frac{X\,ddy}{ds} + \frac{dX\,dy}{ds}$. Substituting these values of $\varphi \sin. u$, $\varphi \cos. u$, $d(\varphi \sin. u)$, $d(\varphi \cos. u)$ in the equations (C): & we will find.

$$2Rpd^2y + pdRdy + Rdydp + \frac{3R X}{ds} \times (dy\,d^2y + dx\,d^2x) + \left(\frac{XdR + RdX}{ds}\right) \times (dx^2 + dy^2) = 0$$

or (on account of $dx\,d^2x + dy\,d^2y = 0$ which is on the supposition that ds is constant; & of $dx^2 + dy^2 = 0$) $2Rpd^2y + pdRdy + Rdydp + Rds\,dX + XdR\,ds = 0$. The terms $2Rpd^2y$ is the same thing as $Rpddy + Rpddy$ & to

$-pds\,dx + Rp\,ddy$, in putting in the first place for R its value $-\frac{ds\,dx}{ddy}$. Consequently our Equation will become $-pds\,dx + Rp\,ddy + p\,dR\,dy + R\,dy\,dp + R\,dX\,ds + X\,dR\,ds = 0$ whose integral is $-ds\int p\,dx + pR\,dy + RX\,ds = A\,ds$. or (in putting for R its value $-\frac{ds\,dx}{ddy}$)
$\int p\,dx + \frac{p\,dx\,dy}{ddy} + \frac{X\,dx\,ds}{ddy} = -A$ or $ddy\int p\,dx + p\,dx\,dy + X\,dx\,ds = -A\,ddy$ whose integral is $dy\int p\,dx + ds\int X\,dx = -A\,dy + B\,ds$. This last equation gives us expunging ds & separating the indeterminates —

$$dy = \frac{(B-\int X\,dx)\,X\,dx}{\sqrt{(\int p\,dx+A)^2 - (B-\int X\,dx)^2}}$$

from where we can always draw the value of y in x terms at least with the assistance of the quadrature of Series —
If the vertical force p is constant & that the perpendicular force $X = x + a$ as in the example of the preceding article; then our equation will become

$$dy = \frac{(2B - x^2 - 2ax)\,X\,dx}{\sqrt{4(px+A)^2 - (2B - x^2 - 2ax)^2}} \cdots$$

XIV.

I will not examine here other hypotheses for the pressures of Voussoirs; I will end with observing that the arch having a convenient figure to the law of the forces which act on it its

parties there will be two extreme Voussoirs that is to say those which are at the Springs Aa, Bb. of the Arch which press the Piers & consequently if those pressures are destroyed, all the System of the Vault will be in equilibrio

XV.

We will go on to the second Problem in Art. IX; A Problem whose Object ~~it is~~ is to determine the law which ought to press the voussoirs when we know the figure of the Vault in such a manner that all its parties be in equilibrio. —

As there are two things to consider in every force its direction & quantity; we will suppose that the directions of the forces which act upon the voussoirs are given & we will seek their quantity

XVI.

Suppose in the first place that the forces φ are in vertical directions. Then ~~since~~ whatever be the figure of the Intrados ACB the equation (C) treated in the same manner as in art XI will give $R\varphi\, dy^2 = A ds^2$; & consequently $\varphi = \dfrac{A ds^2}{R dy^2}$. We will have no more to do then to have φ but to substitute in this expression in place of ds, dy, R, their given values by the nature of the curve **ACB**.

For example, suppose that the Intrados ACB is a Semi-Ellipsis. Surbaissée ou surmontée, whose semi-axis

$OA = a$ & Semi-axis $OC = b$. We will find that

$$\frac{ds^2}{R\,dy^2} = \frac{a^2 b}{(a^2-y^2)\sqrt{a^4-a^2y^2+b^2y^2}}$$ & Then we have

$$\varphi = \frac{A \times a^2 b}{(a^2-y^2)\sqrt{(a^4-a^2y^2+b^2y^2)}}$$

In order to determine the ~~constant~~ quantity A Let us suppose that at the top C, the force φ is represented by a given line m it is clear that we will have $m = \frac{A \times a^3 b}{a^4}$; & consequently $A = \frac{m a^2}{b}$. — —

We see by the general expression of φ that this force increases from the top C even to the Springs A & B where it becomes infinite. Thus the Elliptical arches ought to be loaded very much towards the reins & towards the Springs to be solid.

Fig. 5. If we knew the relations of the forces φ at the top C & at the point M in supposing the angle COM, 45°; we will observe in this hypothesis $\frac{RM}{OM} = \frac{1}{\sqrt{2}}$, or $2(RM)^2 = (OM)^2$; which gives $yy = \frac{a^2b^2}{a^2+b^2}$

Then for the point M we will have $\varphi = \frac{m.(a^2+b^2)^{3/2}}{a\sqrt{(a^4+b^4)}}$ whilst that for the point C we will have $\varphi = m$.

Thus the pressure at the point C or Summit is to the pressure at the point M distant 45° in the ratio of $a\sqrt{(a^4+b^4)} : (a^2+b^2)^{3/2}$

When the Arch is a Semicircle, or when $b = a$ the pressure at the top is half the pressure at

at the point M 45° distant. From whence we see that the great increase of pressure ought to be made from the points of 68° on each side of the vertex upwards ~~even~~ even from the Spring of the Arch. It is then essential to fortify towards the reins Circular & in general elliptical arches. It is there that they generally break —

XVII.

In the second place, let us suppose that the forces Q are perpendicular to the Curve ACB. Whatever be the nature of this curve we will find in general by the method of article XII, $RQ = A$, or $Q = \frac{A}{R}$; which will give afterwards Q in putting for R its value found by the nature of the curve ACB.

For example let as in the preceding article, ACB be a Semi-Ellipsis, whose Semi axis $OA = a$ & semi-axis $OC = b$. We will find $R = \frac{\sqrt{(a^4 - a^2y^2 + b^2y^2)}}{a^4 b}$; & consequently $Q = \frac{A.a^4 b}{\sqrt{(a^4 - a^2y^2 + b^2y^2)}}$. If in order to determine the constant quantity A, we suppose that at the top C the pressure is represented by a given line m, we will have $m = \frac{A.a^4 b}{a^2}$ or $A = \frac{m}{a^2 b}$. —

We see that the pressure at the top C where $y = 0$ is to the pressure at the Springs A, B, where $y = a$ in the relation of $b : a$; & that the pressure at the top C is to the pressure at 45° distant where $y = \frac{a^2 b^2}{a^2 + b^2}$ in the ratio of $\sqrt{(a^4 + b^4)} : a\sqrt{(a^2 + b^2)}$

Thus the pressure of the Voussoirs ought to increase or diminish from the Key Stone to the imposts according as the arch is Surbaissé ou Surmontée. I have no need to add that the pressures throughout are the same when the arch is of full Center ~~or~~ plein ceintre. —

XVIII. —

All these Calculations & examples suffice to show clearly that there is a necessary relation between the figure of the arch & the law of the forces which press the Voussoirs in such a manner that if the construction of the arch is not subjected to this relation there will be no equilibrium between the parties of the System & consequently the arch will break in the weakest place —

XIX. —

We may remark on many Occasions that when the Pier of an arch is found too weak to support the push the vault breaks nearly in two at the middle between each impost & the top. After this Observation Mons.r De la Hire supposed that in the upper half of the arch all the Voussoirs are so adherent between themselves that they can be looked upon alone as one of the same Stone; & that the two inferior parties are solidly bound in all their parts & form as it were one of the same Stone with the Pier corresponding thereto. Thus the Arch can be considered as breaking in the given directions of the joint XZ, X'Z'; The Author sought

Fig.6. as we have already said the effort that the wedge XZCZ'X' exerted against its Piers & the thickness DF which each pier ought to have to be able to resist the force which tended to overturn it. As this Problem which is not difficult is usefull in Practice behold a Solution of it very simple –

XX

Let XZ, X'Z' be any joints of rupture making equal angles with the horizon. Through their middles let be drawn the perpendiculars GQ, GQ, which meet at the point Q upon the axis OC prolonged. Having taken QN to represent the weight of the area X'Z'CZX', let this force be ~~decompo~~ resolved into two others, QS, QI. Imagine that the force QS (we ought to under-stand the same thing of the other side of the arch) is applied to the point G of its direction & repre-sented by Gh = QS; resolve the force Gh into two others Gf, Gq, the one horizontal the other vertical.

The horizontal force Gf tends to overturn the Mason-work XZADFYaX round the point F whilst on the contrary the Masonwork is kept on its base by the force Gq & by its own weight. From the point H the center of gravity of the area XZAa; & from the point R the center of gravity of the rectangle ADFY, let be drawn the vertical lines HK, RL

Let us suppose
- The whole sine or Radius - - - $=1$
- The angle GQO - - - - - - - - - - - - $=m$
- The area XZCZ'X'c - - - - - - - - - $=A$
- The area XZ,Aa - - - - - - - - - $=B$
- AD - - - - - - - - - - - - - - - $=h$
- DF - - - - - - - - - - - - - $=z$
- FK - - - - - - - - - - - - - $=p$
- YP - - - - - - - - - - - - - $=q$
- GP or TF - - - - - - - - - - - $=r$. -

It is clear that we will have the Force QS or Force Gk $= \frac{A.\sin.m}{\sin.2m}$; Force Gf $= \frac{A.\sin.m^2}{\sin.2m}$; Force Gq $= \frac{A.\sin.m.\cos.m}{\sin.2m}$. In considering the moments in relation to the point F we will have the moment of Force Gq $= \frac{A.\sin.m.\cos.m}{\sin.2m} \times r$; the moment of XZADFYAX = moment of XZAa + moment of ADFY $= Bp + \frac{hzz}{2}$. —

Then for the simple state of equilibrium we will have the equation $\frac{A.\sin.m^2 \times (h+q)}{\sin.2m} = \frac{A.\sin.m.\cos.mr}{\sin.2m} + Bp + \frac{hzz}{2}$; from whence we see that the figure & dimensions of the arch being supposed given; also the hight AD of the Pier; as soon as the angle m will be known all the quantities which enter into this equation will be given either immediatly, or in functions of the constant & of the unknown quantities z & m [illegible] the resulting equation will never be more than the second degree. —

XXI.

It happens sometimes that the pier instead of overtur-
-ning all in a piece breaks itself instead of dividing
by horizontal plates. I shall endeavour to find the
figure they ought to have in their exterior part
in this hypothesis: I say exterior part for that which
belongs to the interior is given & forms a plane at least
in the extent AD or BE. A knowing Geometer has resolved
this Problem, in supposing that the portion of the
vault XZCZ'X' exercised only against the Pier a simple
horizontal effort. But really each of the efforts which
result perpendicularly upon the inclined Joints XZ
X'Z' resolve themselves ~~into~~ to two forces the one
vertical the other horizontal; & the first tends to
press the Pier on its base whilst the second tends
to break it or overturn it. —

XXII

Fig. 7 Let there be any portion of an arch as XZCZ'X' which
tends to slide along the two inclined joints XZ, X'Z'.
Through the centers of gravity G, G' of these joints
let be drawn as in Art. XX, the perpendiculars GQ, G'Q
which meet in the prolonged vertical OC. In repre-
-senting the weight of XZCZ'X' by QN, & resolving
this force into two others QS, QI: it is clear that these
two forces are those which act against the piers. I sup-
-pose that the Pier ADFML (the same thing ought
to be understood for the other side of the arch) to be
solidly let into the earth in such a manner that
it cannot slide or overturn itself, but that it

tend to break in every point of its height in such a manner that each section or plate of separation of the parts be horizontal. It remains to find the curve LMF which the exterior face of the Pier ought to form in order that it resist in each part the force that tends to break it. —

Through the point G draw the vertical GH; & prolong the face DA of the Pier till it meet in g the horizontal line Gf. I take for the axis of the curve the vertical gD; & I neglect the Masonry AZGgA; it is clear that if the pier is then sufficiently solid it will be still more so when we put on the masonry which is to be done, since in this last case, the axis of the curve is found evidently placed between the two verticals GH, gD.

XXIII

This being supposed having taken upon the prolonged line QG the part Gh = QS, I resolve the force QS or Gh into two others Gf, Gq the one horizontal the other vertical. Then we draw to the axis gD the ordinate MP which produced meets GH in K & let the ordinate mp be infinitely near.

Let us suppose
- the absolute weight of XZCZ'X'cX — — = P
- the angle GQY — — — — — = m
- gP — — — — — = x
- MP — — — — — = y
- the constant quantity PK — — — — = a
- the density or specific gravity of the matter of which the Pier is composed. — — — = Π

We will have the Force QS or Gh = $\frac{P.\,\text{Sin}.\,m}{\text{Sin}.\,2m}$ & the force

$Gf = \frac{P \sin . m^2}{\sin . 2m}$, Force $Gq = \frac{P \sin . m . \cos . m}{\sin . 2m}$. - - - -

The Piers being supposed divided according to the ordinates PM, pm, must when overturning turn upon the points M, m, these points ought to be looked upon as supports or centers of rotation of the different Levers. The horizontal force Gf tends to produce the overturning of which we speak whilst on the contrary the vertical force Gq. the weight of the part gPM of the Pier & the reciprocal adherence of the two parties gPM, PMFD, concur to keep the part gPM upon its bas PM. Consequently there will be an equilibrium if the moment of the horizontal force Gf. in relation to the point M is less or is not greater than the sum of the moments of the three other forces by relation to the same point.

1. The moment of the horizontal force $Gf = \frac{P \sin . m^2}{\sin . 2m} \times x$.

2. The moment of the vertical force $Gq = \frac{P \sin . m . \cos . m}{\sin . 2m} \times (a+y)$.

3. The distance of the Center of gravity of the Space gPM from the axis gD, is $\frac{\int y^2 dx}{2\int y\, dx}$. Then (in drawing the vertical MR) then the distance of the same point from this vertical is $y - \frac{\int y^2 dx}{2\int y\, dx}$, & consequently the moment of the Space gPM in relation to the point M, is $\Pi y \int y\, dx - \frac{\Pi \int y^2 dx}{2}$. —

4th The reciprocal adherence of the two parties gPM, PMFD. being supposed proportional to the extent of the surface PM by which they touch it is clear that the moment of this force in relation to the point M will be proportional to $y \times \frac{y}{2}$. Thus supposing that in a given length h the adherent force is = any weight known weight Q the moment in.

question will be represented by $\frac{Qy^2}{2h}$.

Consequently / in taking any arbitrary number k such that its least value be not below 1, / we will have the fundamental Equation. —

$$\frac{k.P\sin.m^2.x}{\sin.2m} = \frac{(a+y)P\sin.m.\cos.m}{\sin.2m} + \Pi y\int y\,dx - \frac{\Pi\int y^2dx}{2} + \frac{Qy^2}{2h};$$

from which it is necessary to draw the relation between x & y. To do which je différencie the two members. Which gives

$$\frac{k.P\sin.m^2.dx}{\sin.2m} = \frac{P\sin.m.\cos.m.dy}{\sin.2m} + \Pi\,dy\int y\,dx + \frac{\Pi y^2dx}{2} + \frac{Qy\,dy}{h};$$

Différenciant encore in supposing dy constant, we will have

$$\frac{kP\sin.m^2.ddx}{\sin.2m} = 2\Pi y\,dx\,dy + \frac{\Pi yy\,ddx}{2} + \frac{Q\,dy^2}{h};$$ Let $dx = z\,dy$

& consequently $ddx = dz\,dy$; we will have $\frac{kP\sin.m^2.dz}{\sin.2m} = 2\Pi z\,y\,dy + \frac{\Pi yy\,dz}{2} + \frac{Q\,dy}{h}$ or rather $\left(\frac{kP\sin.m^2}{\sin.2m} - \frac{\Pi yy}{2}\right)$ $dz = 2\Pi\,zy\,dy + \frac{Q\,dy}{h}$. —

I make $\frac{kP\sin.m^2}{\sin.2m} - \frac{\Pi yy}{2} = u^2$; & consequently

$$2\Pi y\,dy = -4u\,du;\ dy = -\frac{4u\,du}{2\Pi y} = -\frac{u\,du\sqrt{\frac{2}{\Pi}}}{\sqrt{\left(\frac{kP\sin.m^2}{\sin.2m} - u^2\right)}}.$$ —

From whence it follows that in taking —

$$\frac{Qu\sqrt{\frac{2}{\Pi}}}{h\sqrt{\left(\frac{kP\sin.m^2}{\sin.2m} - u^2\right)}} = V.\ \underline{\text{fonction de } u},$$ we will have the

transformed $uu\,dz + 4zu\,du + V\,du = 0$; (or in multiplying by u^2) $u^4dz + 4zu^3du + Vu^2du = 0$ Whose integral is

$$zu^4 + \int Vu^2du = A;$$

or else / in returning for V its value, making in order to abridge it $\frac{Q}{h}\sqrt{\frac{2}{\Pi}} = a$ & $\frac{kP\sin.m^2}{\sin.2m} = b^2$ & effecting the entegration indiqué $\int Vu^2du$) —

$$n u^4 - n b^2 \sqrt{b^2-u^2}) + \frac{u(b^2-u^2)^{\frac{3}{2}}}{3} = A. -$$

Expunging ~~the~~ Eliminant ξ & u & making to abrige it $\frac{2b^2}{\Pi} = c^2$, $n b^2 \sqrt{\frac{\Pi}{2}} = p^3$; $\frac{n}{8}\left(\frac{\Pi}{2}\right)^{\frac{3}{2}} = q$; we will have —

$$\frac{\Pi^2}{4} \times (c^2-y^2)^2 \times \frac{dx}{dy} - p^3 y + q y^3 = A;$$ or else

$$dx - \frac{4p^3}{\Pi^2} \times \frac{y\,dy}{(c^2-y^2)} + \frac{4q}{\Pi^2} \times \frac{y^3\,dy}{(c^2-y^2)} = \frac{4A}{\Pi^2} \times \frac{dy}{(c^2-y^2)};$$

Whose integral is

$$x - \frac{2p^3}{\Pi^2(c^2-y^2)} + \frac{4q}{\Pi^2}\left(\frac{c^2}{2(c^2-y^2)} + L\,\frac{\sqrt{c^2-y^2}}{c}\right) =$$

$$\frac{4A}{\Pi^2}\left(\frac{y}{2c^2(c^2-y^2)} + \frac{1}{2c^3}\,L\,\frac{c+y}{\sqrt{c^2-y^2}}\right) + C;$$ The Equation of the curve LMF in finite termes. —

The Constant quantities A & B ought to be deter-mined by these two conditions: 1 that x being = nothing y will also be nothing or a given quantity; the beginning of the curve will be at G, or upon a given point of the horizontall Gx. 2 That ~~the~~ at the beginning we have. $\frac{dx}{dy} = \frac{Cof.\,m}{Sin.\,m}$ because then there only remains the two forces Gf, Gq & that their resultant ought to be parallel to the first area or element of the curve.

Knowing thus the nature of the curve LMF we will know the last Ordinate DF, or the thickness of the Pier at its bass. We will know besides the thickness that answers to every other point of its height

Section Second.—

of the equilibrium of Voutes en dôme.—

We have almost nothing writt upon the equilibrium of Arches of a dome. I know none on this subject but an excellent memorial by M. Bouguer printed amongst those of the academy (of year 1734). It has for its title: Upon curve lines that are necessary to form vaultes of a dome. The Author there makes it appear that there exists an infinite number of curve lines that can be employed to form the vault which is required; & at the same time he teaches us to make a choice among all these curves which is the most advantageous. My object which is to determine the dimensions that ought to be given to the pieddroits or Piers of the vault of a dome for the equilibrium has nothing in common with the work of M. Bouguer.——

II

Let there be a dome produced by the revolution of the Space comprehended between the curve of the intrados ACB & the curve of the extrados acb around the montée or axis OCc. Let us suppose this dome tends to break according to the directions of the inclined Joints XZ, X'Z', (Fig. 8) which tend towards the axis of revolution; in such a manner that the superior part ZCZ'X'cX can be looked upon as one & the same body; & that the part AZXa (it is the same for the other side of the vault) let it be looked upon as a body in its parts & moreover

form as it were the same Mason work with the corresponding Pier ADF"T. I go to find the thickness that ought to be given to the Pier in the hypothesis where the masontry of which we speak cannot break in any point of its height but only overturns in turning upon the point F of its base.-

III.

Let us imagine that through the axis YCc of the vault there passes two planes YLiaCc, YlabcC which make an angle infinitly little & which being prolonged to a convenient length determines on either part of the Axis, two Onglets egaux & corresponding in the superior partes of the dome & two Onglets egaus aussi & corresponding in the inferior parts. The plan YDFTaALc is looked on as divided into two partes, the angle of the two first planes, & each paire d'onglets correspondans.
Through the centers of gravity G & G' of the two inclined planes XZ, X'Z', let be drawn perpendicularly to the planes the right lines GQ, G'Q which meets the Axis of the dome at the point Q; & conceive that the gravity of a System of two Onglets which answer to the Superior part ZCZ'X'cX be reunited at the point Q. Let this force be represented by QN: & resolve it into the two others QS, QI directed according to the right lines QG, QG'. These forces QS, QI are the efforts that the sistem of the two Superior Onglets exercer or exert against the sistem of the two inferior Onglets.-

IV. —

Conceive that the force QS is applied to the point

G of its direction & represented by $Gh = QS$; afterwards resolve this force Gh into two others Gf, Gq, the one horizontal the other vertical. It is clear (in prolonging Gf, Gq, FT) that the force Gf, in acting at the extremity of the arm of the lever FX, tends to overturn round the point F or rather round the right line Ll, l'onglet represented by the mean section $ZADF$ TaX; & on the contrary the force Gq, in acting at the extremity of the arm of the lever FH, conspires with the weight of l'onglet which we came to show to firm this same l'onglet on it base. The question is then only to in the equation the equilibrium after ~~this~~ consideration —

In order to abrige the Calculation the portion of the d'onglet, represented by its profile $AaXZ$, I will substitute an onglet of the same base & same height represented by its rectangular profile $Aax\lambda$. This substitution is as much more permitted as the l'onglet partiel $AaXZ$ z, by relation to the point F, an arm of a lever. a little longer than that of the onglet $Aax\lambda$. It is true that the ratio of the diminution of the thickness of the vault en montant, the second l'onglet is a little greater than the first; but generally the compensation is not compleat. As to the rest nothing will be more easy than to treat rigourously this point of the question in each particular case if we juge it a propos. —

V.

When we draw from the indetermined point R of the axis the right lines RP, Rp, Rr, in the vertical plane moyen des Onglett, & in the two vertical planes which terminate them. Let there be drawn afterwards parallel to the right lines mn, pr, the neighbouring right line infinitly near ey, dg. — —

Suppose {
the radius or whole Sine - - - - - - - - = 1
the angle OQG - - - - - - - - - - = m
the angle LYC, or pRr - - - - - - - - = ϖ
AO - - - - - - - - - - - - - = a
Aa - - - - - - - - - - - - - = c
Aλ - - - - - - - - - - - - - = g
AD - - - - - - - - - - - - - = h
AV or DH - - - - - - - - - - - = l
Rz - - - - - - - - - - - - - = y
le double onglet produced by the revolution of the Area ZCZ'X'CX about Cc - - - - - - - = 2ϖ.S,
(S being a given quantity by the figure of the Voûte
the unknown thickness DF du pied droit et Pied - - - - - - - - - - - - - - - = z
}

It is evident that ~~the force~~ we will have the Force $Gh = \frac{2\varpi.S.\sin.m}{\sin.2m}$; Force $Gf = \frac{2\varpi.S.\sin.m^2}{\sin.2m} = \varpi.S.\text{tang}.m$;
Force $Gq = \frac{2\varpi.S.\sin.m.\text{Cos}.m}{\sin.2m} = \varpi.S.$ - -

And if we consider the moments in relation to the point F we will have moment of force $Gf = \varpi.S.\text{tang}.m \times (h+g)$;
moment of force $Gq = \varpi.S.\times(l+z)$.

The little trapezzium $eygd = zi \times ey = dy \times y\varpi$: & its

moment in relation to the point π', is $\omega y\, dy (a+z-y)$. whose integral is $\omega.\frac{(a+z)y^2}{2} - \frac{\omega y^3}{3} + A.$ --

The integral ought to vanish when $y = a$, & receive its compleat value, when $y = a+z$. Thus the moment of the area $m\,p\,r\,n = \omega\left[\frac{(a+z)^3}{6} - \frac{a^2 z}{2} - \frac{a^3}{6}\right]$; & consequently the moment de l'onglet, represented by the profile ADFT, will be $= \omega h\left[\frac{(a+z)^3}{6} - \frac{a^2 z}{2} - \frac{a^3}{6}\right]$. .

We will find in the same manner that the moment de l'onglet represented by the profile $A\lambda x a$, has for its expression $\varpi g\left(\frac{2acz + c^2 z - ac^2}{2} - \frac{c^3}{3}\right)$ --

This being supposed there will be an equilibrium if the moment of the force Gf is less or is not greater than the sum of the moments of the force Gq & of the two l'onglets ADFT, $A\lambda x a$. Thus taking the arbitrary number k which must not be below 1 we will have the equation -

$$k\varpi.S.\tan g.m.(h+g) = \varpi.S.(l+z) + \varpi h\left(\frac{(a+z)^3}{6} - \frac{a^2 z}{2} - \frac{a^3}{6}\right) + \varpi g\left(\frac{2acz + c^2 z - ac^2}{2} - \frac{c^3}{3}\right) \text{ or else -}$$

$$(A)\quad z^3 + 3az^2 + \left.\begin{matrix}\frac{g}{h}(6ac + 3c^2) \\ + \frac{6S}{h}\end{matrix}\right\} z \left.\begin{matrix} + \frac{6S.l}{h} \\ - \frac{g}{h}(3ac^2 + 2c^3) \\ - \frac{6kS.\tan g.m(h+g)}{h}\end{matrix}\right\} = 0$$

an equation of the third degree which will give the value of the unknown quantity z which is the thickness DF or AT of the Piere ou pied-droit. ——

VI.

It often happens that the dôme carries at its top a kind of lantern which makes a considerable load. Then its is necessary at first to determine by a detail of the parties of this lanterne, the whole weight that results therefrom: afterwards, we will convert this mass into a cylinder V

Fig. 9. concentric with the dôme of the same matter as it & having a determined base & height. Let r = rad. of the base of this cylinder V, f = its height, Π the relation of its circumference to diat. the cylinder V will have for its value $\Pi r^2 f$ & consequently le double onglet, corresponding au double onglet $2\varpi S$, will have for its expression $\frac{2\varpi . \Pi r^2 f}{\Pi}$, or $2\varpi . r^2 f$, as being the fourth term of a proportion of which the three first are Π, 2ϖ, $\Pi r^2 f$. The Equation (A) will apply itself to this case in putting in place of S, $S + r^2 f$. —

VII.

The upper part des pied-droits is commonly crowned with an attic whose mass joined to that of the other constructions, can be looked upon as forming a tour represented by the rectangular profile $A\varpi$. Now it is evident that in consequence of this increase of pressure on the pied-droit, its thickness DF resulting from the equation (A) may be diminished & reduced to another thickness Df. We will make enter into the calculation l'onglet represented by $A\omega$ precisely in the same manner as we have done with l'onglet represented by Ax (fig 8) —

VIII.

It follows from the three preceding Articles that

in supposing $A\alpha = p$, $\alpha\varpi = q$, $Df = u$; considering $p\ q$ as
Fig. 9 given quantities; & putting $S + r^2 f$ instead of S in the equation (A), we will have here –

$$(B)\quad \left.\begin{matrix} u^3 + 3au^2 + \dfrac{6(S+r^2f)}{h} \\ + \dfrac{g(6ac+3c^2)}{h} \\ + \dfrac{q(6\varpi p + 3p^2)}{h} \end{matrix}\right\} u \left.\begin{matrix} + \dfrac{6(S+r^2f^2).l}{h} \\ - \dfrac{g(3ac^2+2c^3)}{h} \\ - \dfrac{q(3\varpi p^2 + 2p^3)}{h} \\ - \dfrac{6k(S+r^2f)\,\text{tang.}\,m(h+g)}{h} \end{matrix}\right\} = 0$$

which contains the solution of the Problem when taking in the weight of the lanterns & that of the Attic. –

IX.

As it is maître to increase more or less the masonry $A\varpi$, we can suppose that the base $A\alpha = Df$, & that its height $\alpha\varpi$ is given. Making then $Df = A\alpha = t$, $\alpha\varpi = q$; putting $S + r^2 f$ for S; & observing that the height du pied-droit is just now $= h + q$ (which I make $= H$ to abrige it) whilst all the rest remains the same as before; we will find, by art V.

$$(C)\quad \left.\begin{matrix} t^3 + 3at^2 + \dfrac{6(S+r^2f)}{H} \\ + \dfrac{g(6ac+3c^2)}{H} \end{matrix}\right\} \times t \left.\begin{matrix} + \dfrac{6(S+r^2f).l}{H} \\ - \dfrac{g(3ac^2+2c^3)}{H} \\ - \dfrac{6k(S+r^2f)\,\text{tang.}\,m.(h+g)}{H} \end{matrix}\right\} = 0$$

an equation of which we can make the greatest use in practice. –

X

Seeing just now how the quantity S ought to be determined. We will now resolve this Problem in betting

upon

the curves of the <u>intrados or extrados</u>, as Ellipses or Parabolas which is the two most ordinary cases. - -

Let at first ACB, acb be two Semi-Ellipses. Suppose the semi-axis $OA = a$; the semi axis $OC = b$; the abscissa $CP = x$ the corresponding ordinate $ZP = y$; the ratio of the circumference to the diat. $= \Pi$. we will have $y^2 = \frac{a^2}{b^2}(2bx - x^2)$. The expression for the Area of a Solid which will be produced by an entire revolution of the Segment CPZ about CP, is $\frac{\Pi a^2}{b^2}(2bx - x^2) \times dx$ whose integral is $\frac{\Pi a^2 x^2}{b^2}\left(b - \frac{x}{3}\right)$. This integral is then the Value of the Solid CZZ'. -

In like manner if we make $Oa = a'$; $OC = b'$; $cP = x'$ we will find Solid $cxx' = \frac{\Pi a'^2 x'^2}{b'^2}\left(b' - \frac{x'}{3}\right)$. -

Now it is evident if we divide by Π, the difference of two Solids of which we come to speak, the quotient will surpass the quantity which has been named S in general (art V) as to the little Solid which will be described by the triangle <u>mixtiligne</u> XZx & αX which we can omit it without any sensible error. Thus.

$$S = \frac{a'^2 x'^2}{b'^2}\left(b' - \frac{x'}{3}\right) - \frac{a^2 x^2}{b^2}\left(b - \frac{x}{3}\right) \text{ sensibly or ultimatly}$$

The joint XZ being perpindicular to the curve ACB the angle which has been called m (art V) is equal or perhaps may be deemed equal to that which the tangent to the ellipsis at the point Z makes with the vertical. From whence it evidently follows that $\tan g.\, m = \frac{\text{ordinate}}{\text{Subtangent}} = \frac{a(b-x)}{b\sqrt{2bx - x^2}}$

Just now it is necessary to determine upon the hypothesis on the Subject of the point of rupture Z of the dome

Let us suppose that the point Z is the intersection of the curve ACB with the diagonal OK of the rectangle OCKA; which agrees sufficiently with experience. Then we will have this proportion viz $OC(b) : CK(a) :: OP(b-x) : PZ\left(\frac{a}{b}\right) \times \sqrt{2bx - x^2}$; & consequently $b - x = \sqrt{2bx - x^2}$ that which gives $x = b\left(1 - \frac{1}{\sqrt{2}}\right)$, & $y = \frac{a}{b}\sqrt{2bx - x^2} = \frac{a}{\sqrt{2}}$. Knowing x we will have x', since $x' = Cc + x = b' - b + b\left(1 - \frac{1}{\sqrt{2}}\right) = b' - \frac{b}{\sqrt{2}}$.

The quantity which has been called g (art V) may perhaps called equal to OP; & that which has been called l. may perhaps be deemed $= AO - ZP$. Thus, we have $g = \frac{b}{\sqrt{2}}$; $l = a\left(1 - \frac{1}{\sqrt{2}}\right)$.

Substituting all these values in the equation (C) there will remain only the unknown quantity t. —

XI. — —

When the curves ACB, acb are Parabolas, if every thing else remain the same, we may call p Parameter of Parabola ACB, which is $= \frac{a^2}{b}$; p' that of the parabola acb which is $= \frac{a'^2}{b}$; we will have at first the Solid: $CZZ' = \frac{\pi p x^2}{2}$; the Solid $Cxx' = \frac{\pi p' x'^2}{2}$; $S = \frac{p'x'^2 - px^2}{2}$; Tang. $m = \frac{\sqrt{px}}{2x}$ Afterwards $x = b\left(\frac{3 - \sqrt{5}}{2}\right)$; $x' = b' - b + \frac{b(3 - \sqrt{5})}{2} = b' + \frac{b(1 - \sqrt{5})}{2}$; $g = b - x = b\left(\frac{\sqrt{5} - 1}{2}\right)$; $l = a - \sqrt{px} = a\left[1 - \left(\frac{3 - \sqrt{5}}{2}\right)\right]$; from whence we see that in the equation (C) all will be in like manner known except t. — —

XII.

We will now make an application of this Theory. I will take for example, the dôme of the Church of Sainte Geneviève of Paris, constructed by M. Soufflot. —

In this dôme, the curves ACB, acb are Parabolas; & if we draw the chords AC, CB, the triangle ACB is equilateral. Each Side of that triangle =*64 feet, which gives the perpendicular OC = 55·424 feet, very nearly. The height AD du pied droit = 44 feet; the reduced height Ad of the Attic = 20 f. the thickness Aa of the Vault at its Spring = 3 feet; & the thickness Cc at the top = 1 foot 6 Inches. The lanterne reduced to the cylinder V which has for its base a circle 15 feet diam. & height 10 feet

After these being given, we will find (in combining together figures 8 & 9. & taking all the measures in linear feet, squared or cubed) $a = 32$; $b = 55·424$; $a' = 35$; $b' = 56·924$; $p = \frac{a^2}{b} = 18·476$, $p' = \frac{a'^2}{b'} = 21·520$; $x = 21·172$; $x' = 22·672$; $g = 34·252$; $l = 12·222$; $h = 44$; $H = 64$; $c = 3$; $2r = 15$; $f = 10$; $S' = 1390$; $r^2 f = 562·5$; Let $k = 1$. In substituting all these values in the general equation (C) of art. IX, it will become

$t^3 + 96t^2 + 505·608\,t - 4943.128 = 0$

Now the Value of t which answers this equation is very nearly $t = 4·93$. The other two roots of the equation are negative & unnecessary to consider. Thus the thickness of the pied-droit ought to be 4 ft 11 Inches or

thereabout for the simple state of equilibrium. —

We may observe that if we placed the point of rupture L a little higher or a little lower than we have done there would result no sensible change in the value of t. It is that which we will easily see by examining in general the effects which tend to produce the two forces Gf, Gq (fig. 8) Besides we cannot assure ourselves directly by the calculation

M. Soufflot gives 5 feet 8 inches of thickness to his pied-droit in the weakest parts, & 16 feet of thickness in the four principal parts which answer to the ~~Centers~~ des piliers destined to carry the dôme. From whence we see that the pied droits will have a strength more than sufficient to support the push of the dôme & consequently there is no doubt but this vault will be sufficiently solid —

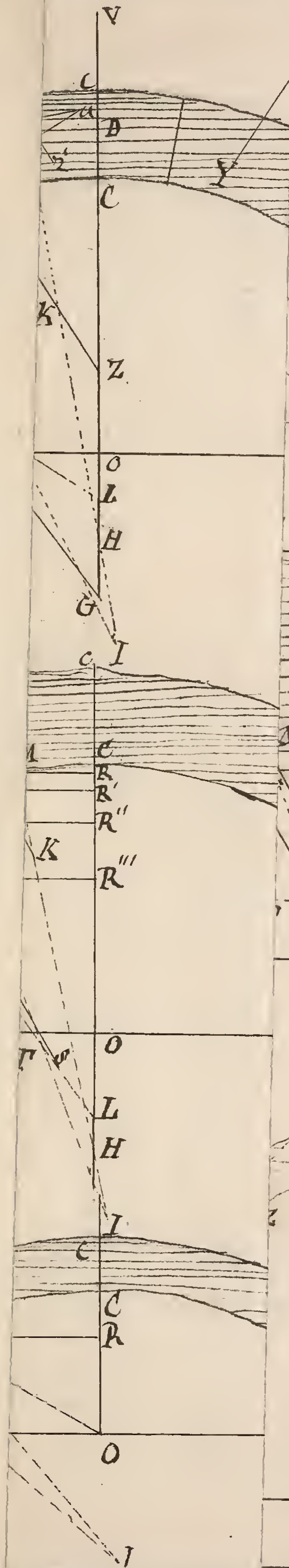
If all the other dimensions besides remained the same we might give 4 feet of thickness to the spring Aa of the vault, 2 feet to the top Cc, & 20 feet of height to the cylinder V which represents the weight of the lanterne; that is to say if all the other dimensions remained unaltered we made —

$c=4$; $a'=36$; $b'=57.424$; $f=20$: we would find the equation $t^3+96t^2+721.879t-7826.051=0$ which gives nearly $t=5.91$, or $t=5$ feet 10 inches 11 lines nearly. —

The thickness that M. Soufflet gives to the pied-droits of his dôme will be as yet very sufficient for this case; having regard to the binding produced by the mortar in every part of the masonry, & to the thickness of 16 feet that the pied-droits will have in their four principal points of support. —

XIII. -

Scholium We might ~~treat~~ here in the subject of the Vaults of a dôme, a Problem analagous to that which has been resolved for the Vaultes en berceau, in art. 23 of the preceding Section; but as in practice there is no case where we can subject the exterior form of a tour or turning which supports a dôme to that which the calculation would require I will refrain from this research.



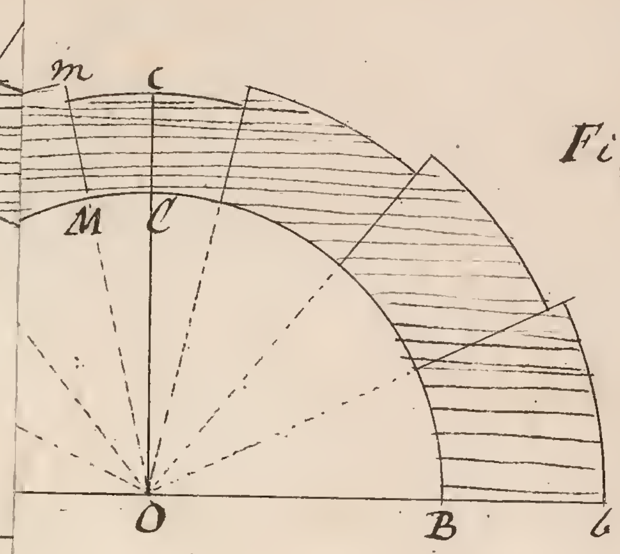


Fig. 2.

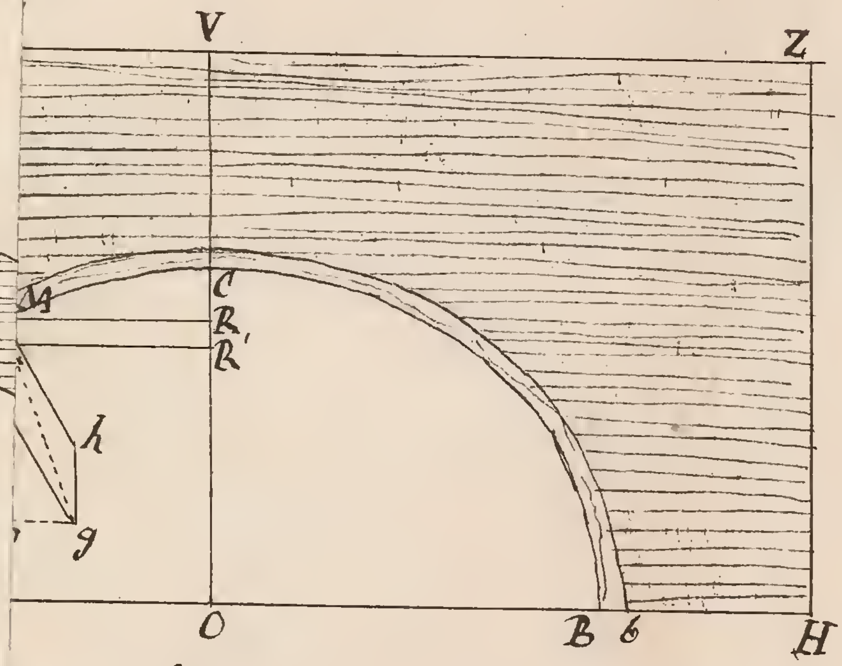


Fig. 4.

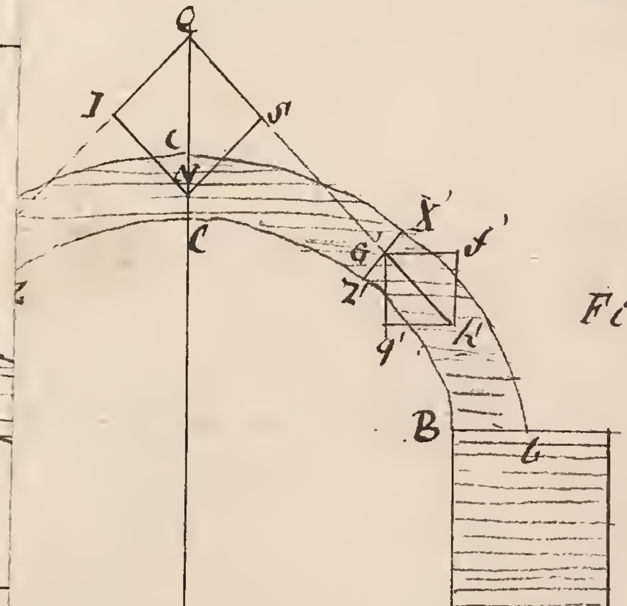
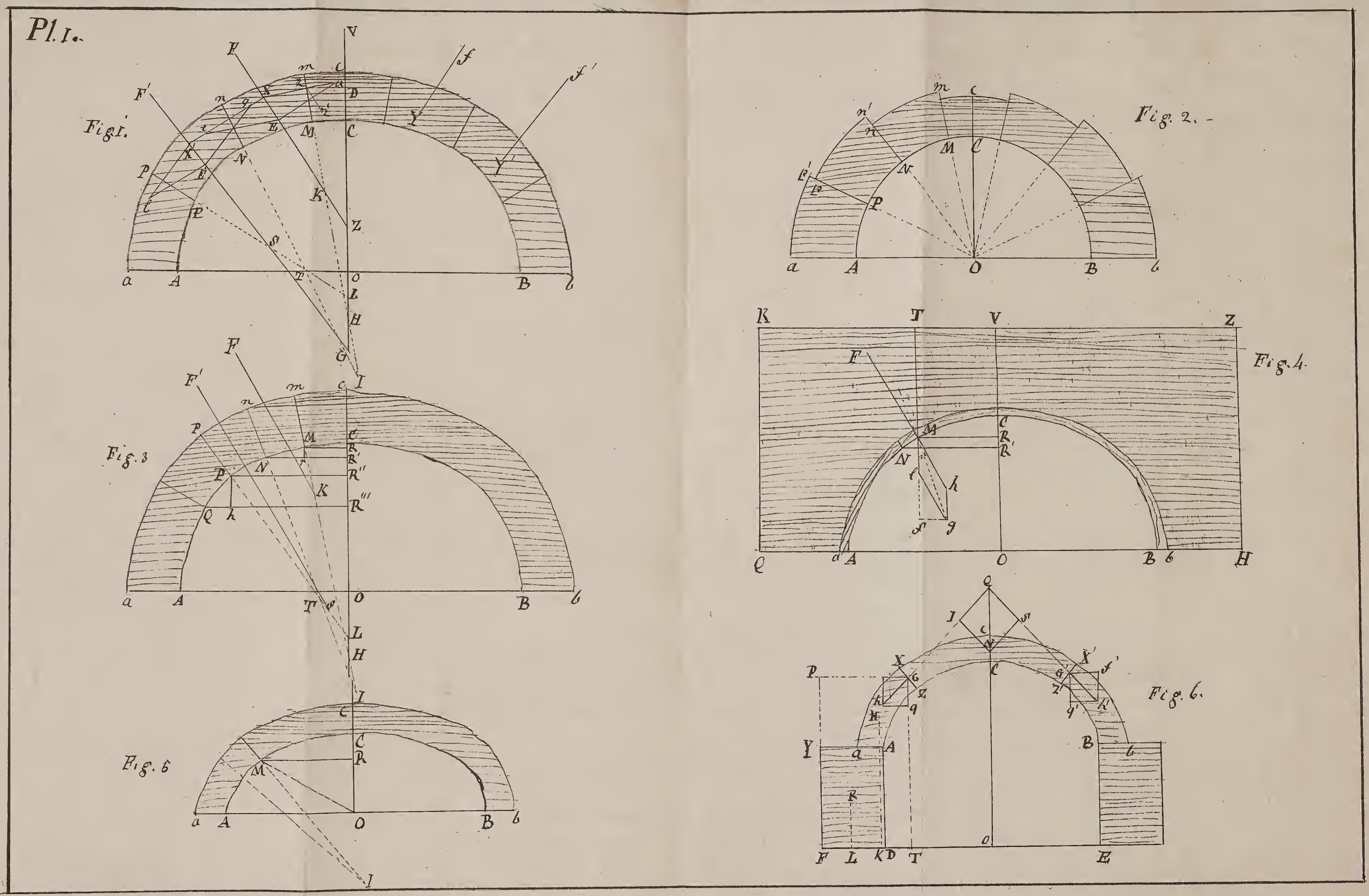


Fig. 6.


Pl. I.
Fig. 1.
Fig. 2.
Fig. 3
Fig. 4.
Fig. 5
Fig. 6.

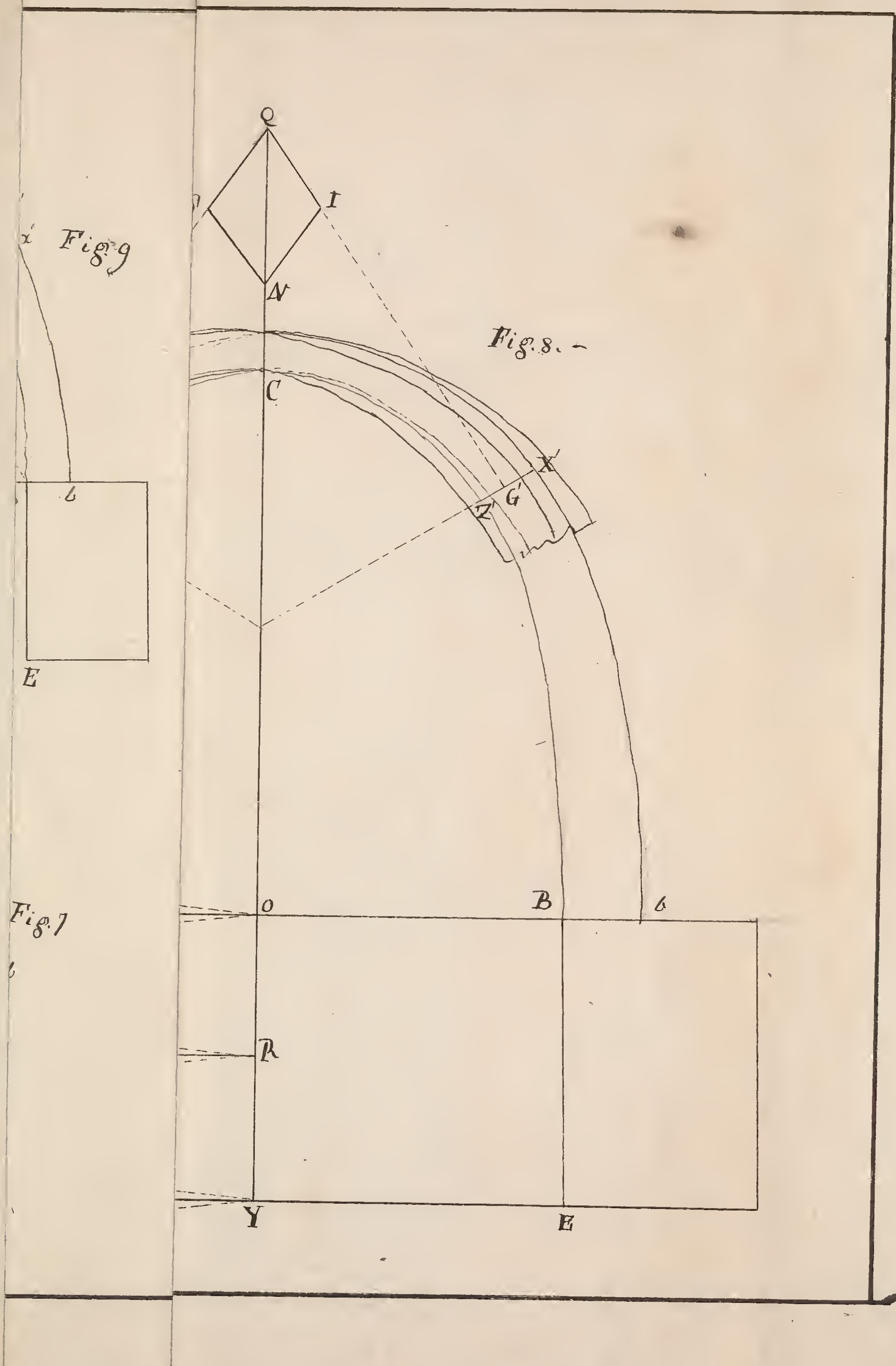


Fig. 9
Fig. 8. –
Fig. 7
Q
I
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C
X'
G'
Z
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o
B
b
R
Y
E

Pl. 11.

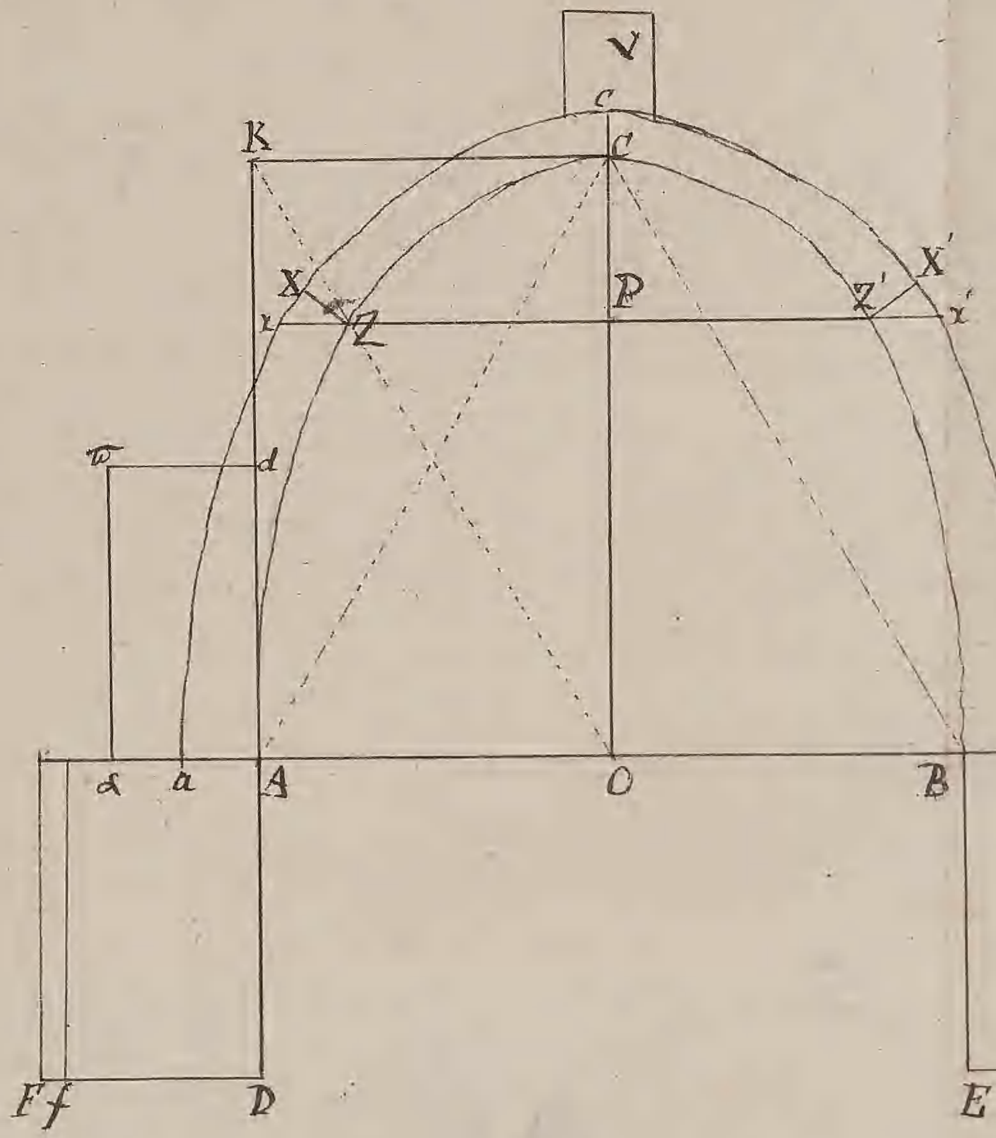

Fig. 9

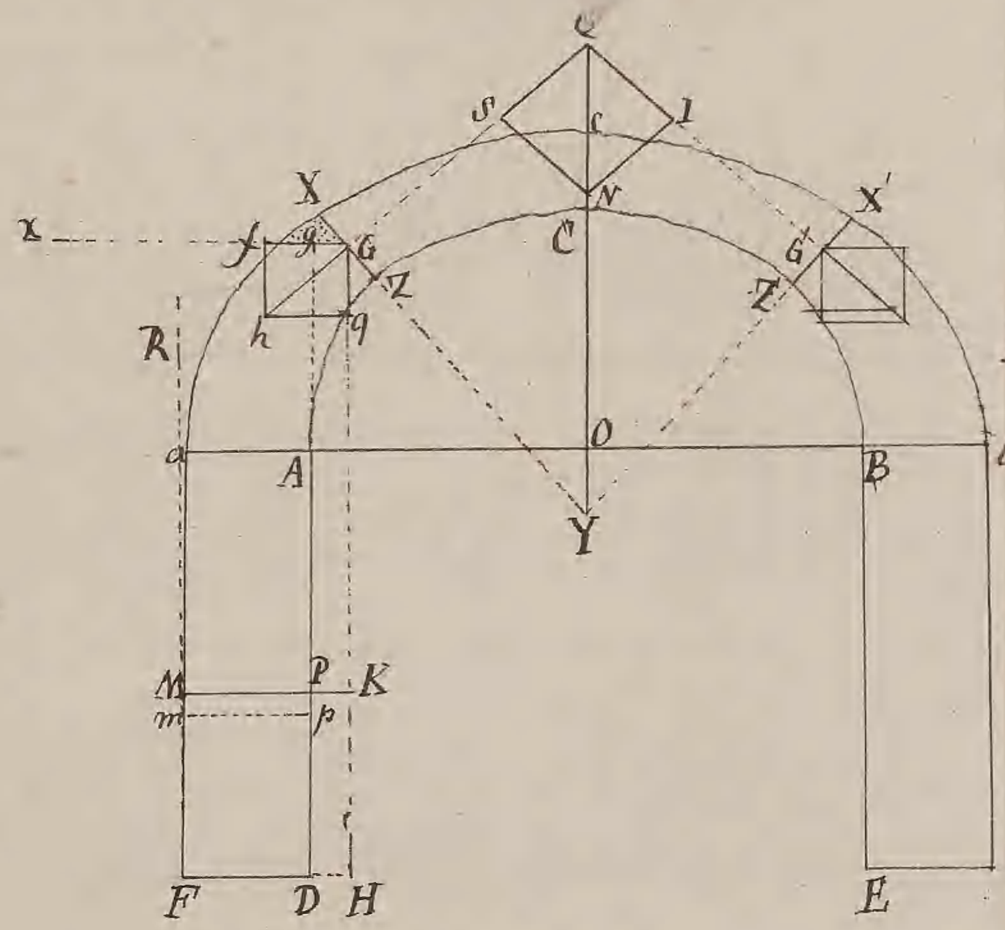

Fig. 7

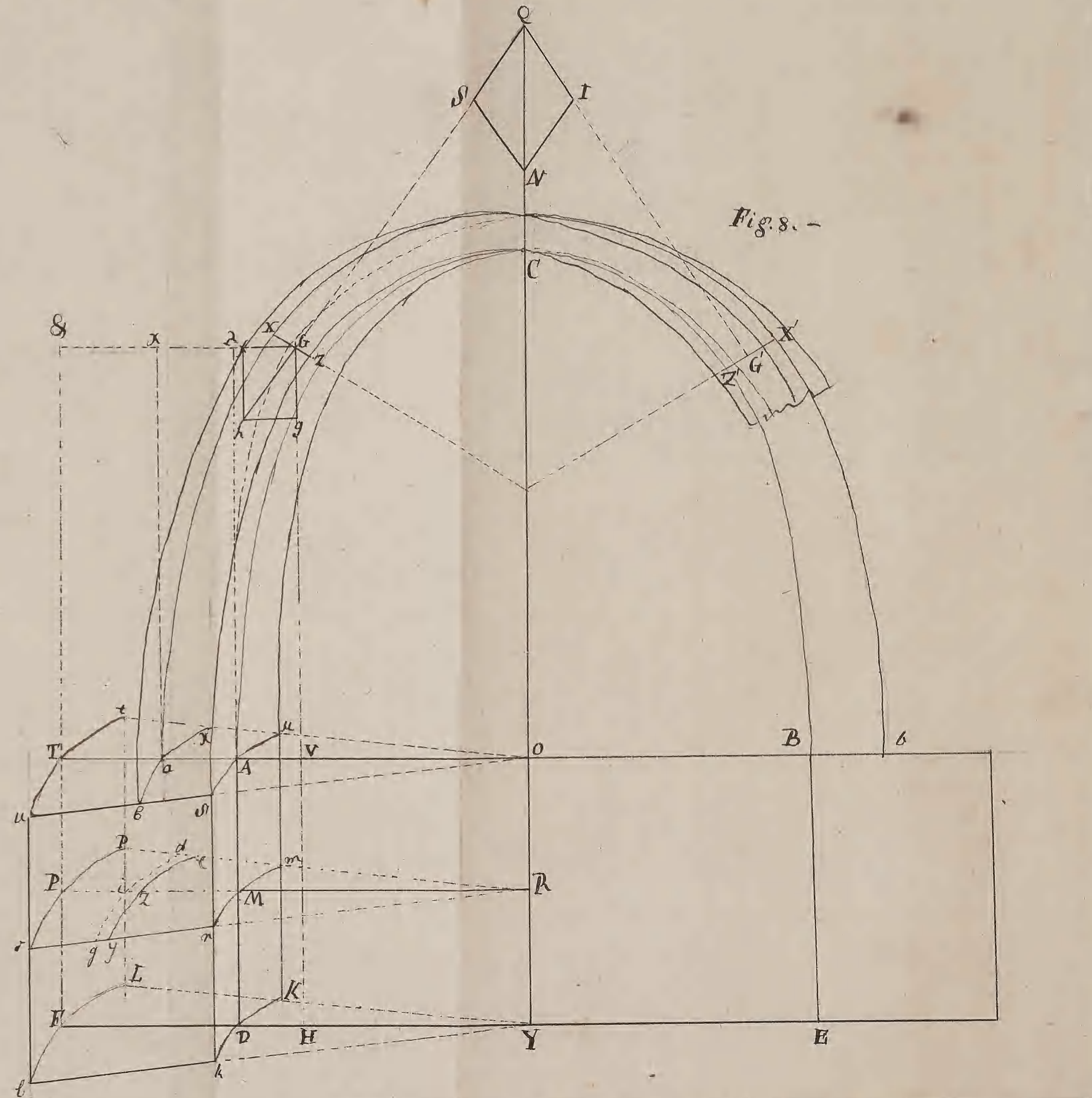

Fig. 8.

Printed at Paris in the 1783

Description des projets de la construction des Ponts de Neuilly, de Mante d'Oleans & autres; du projet du Canal de Bourgogne pour la communication de deux Mers par Dijon; & celui de la conduite des eaux de l'Yvette & de la Brevere à Paris. Ex. Soixante-Sept. Planchés. Approuvé par l'Academie Royale des Sciences. Dedie au Roi. Par M. Perronet. Chevalier de l'Order du Roi, son Architecte, & premier Ingenieur pour les Ponts & Chaussées des Academies Royales des Sciences de Paris. Stockholm &c &c, A Paris, de l'Imprim. Royale. 2 Vol. in folio maximo. 153 liv. brochées en cartons: & se trouve chez Jombert. rue Dauphine

An Experimental Examination of the Quantity and Proportion of Mechanic Power necessary to be employed in giving different Degrees of Velocity to heavy Bodies from a State of Rest. By Mr. John Smeaton, F. R. S.

About the year 1686: Sir Isaac Newton first published his Principia, and conformably to the language of Mathematicians of those times defined, that "the quantity of "Motion is the measure of the same, arising from the Velocity and "quantity of Matter conjointly." Very soon after this Publication, the truth or propriety of this Definition was disputed by certain Philosophers, who contended, that the measure of the quantity of Motion should be estimated by taking the quantity of Matter and the square of the Velocity Conjointly. There is nothing more certain, than that from equal impelling Powers, acting for equal intervals of time, equal increases of Velocity are aquired by given Bodies, when unresisted by a Medium: thus gravity causes a Body, in obeying its impulse, during one Second of Time, to aquire a Velocity which would carrie it uniformly forward, without any additional impulse, at the Rate of

of 32 ft. 2 in. per Second; and if Gravity is suffred to act upon it for 2 Seconds, it will have, in that time, aquired a Velocity that would cary it at an uniform Rate, just double of the former; that is, at the Rate of 64 ft. 4 in. per Second. Now, if in consequence of this equal increase of Velocity, in an equal increase of time, by the conti=nuance of the same impelling Power, we define that to be a Double quan=tity of Motion, which is generated in a given quantity of Matter, by the action of the same impelling Power for a double time; this will be Co=in=cident with Sir Isaac Newton's definition above mentioned; where=as, in trying experiments, upon the total effects of Bodies in Motion it appears, that when a Body is put in Motion, by whatever cause the im=pression it will make upon an uniformly resisting Medium, or upon uniformly yeilding Substances, will be as the Mass of Matter of the moving Body, multiplied by the Square of its Velocity; the question, therefore, properly is, wether those terms, the Quantity of Motion the Momenta, of Bodies in Motion, or Forces of Bodies in Motion, which have generaly been esteemed Synonymous, are with the most propriety of language to be esteemed, equal, Double, or Triple when they have been generated by an equable impulse, acting for an equal, Double, or Triple time: or that it should be measured by the effects of being Equal, Double, or Triple in overcoming Resistances.

before a Body in Motion can be stopped. For, according as those terms are understood in this or that way, it will necessarily follow, that the Momenta of equal Bodies will be as the Velocity's or as the Squares of the Velocities respectively; it being certain, that, whichever we take for the proper Definition of the term quantity of Motion, by paying a proper regard to the collateral circumstances that attend the application of it, the same Conclusion, in point of Computation, will result.

I should not, therefore, have thought it worth while to trouble the Society upon this subject, had I not found, that not only myself and other practical Artists, but also some of the most approved writers, had been liable to fall into errors, in applying these Doctrines to practical Mechanics, by somtimes forgetting or neglecting the due regard which ought to be had to these collateral circumstances.

Some of these errors are not only very considerable in themselves, but also of great consequence to the Public, as they tend greatly to mislead the practical Artist in works that occur daily, and which often require very great sums of money in their execution.

I shall mention the following instances.

Desaguliers, in his second volume of Experimental Philosophy, treating upon the question concerning the forces of Bodies in Motion, after taking much pains to shew that the dispute, which had then subsisted fifty years, was a dispute about the

the meaning of words: and that the same Conclusion will be brought out, when things are rightly understood, either upon the old or new oppinion, as he distinguishes them; among other things, tells us, that the old and new oppinion may be easily reconciled in this instance: that the Wheel of an undershot Water Mill is capable of doing quadruple work when the Velocity of the water is Doubled, instead of Double work only; " Because, (the "adjutage being the same,) says he, we find, that as the waters "velocity is double, there are twice the number of particles of "water that issue out; and therefore the Ladleboard is struck "by twice the Matter, which Matter moving with twice the ve= "=locity that it had in the first case, the whole effect must be "quadruple, though the instantaneous stroke of each Particle "is increased only in a simple proportion of the Velocity." See Voll. 2d Annotations on lecture 6th Page 92.

Again in the same vollume, lecture 12th Page 424 referring to what went before, he tells us, " The knowledge of the forgoing "particulars is absolutely necessary for setting an Undershot wheel " to work; but the advantage to be reaped from it would be still " guess work, and we should still be at a loss to find out the " utmost it can perform, if we had not an ingenious Proposition " of that mechannic M. Parent. of the Royal Academy

of Sciences, who has given us a Maximum in this case, by shewing, that an Undershot Wheel can do the most work, when its velocity is equal to the third part of the velocity of the water that drives her, &c. because then two thirds of the water is employed in driving the wheel with a force proportionable to the square of its velocity. If we multiply the surface of the adjutage or opening by the height of the water, we shall have the column of water that drives the wheel. The wheel thus moved will sustain on the opposite side only four ninths of that weight, which will keep it in equilibrio; but what it can move with the velocity it goes with, will be but one third of that weight of equilibrium; that is $\frac{4}{27}$ths of the weight of the first column, &c. — This is the utmost that can be expected."

The same conclusion is likewise adopted by Mc Laurin, in art. 907. Page 728. of his Fluxions, where giving the fluxionary deduction of M. Parents proposition, he says, "that if A represents the weight which would ballance the force of the stream, when its velocity is a; and U represents the part of the engine, which it strikes when the motion of the Machine is uniform &c. — The Machine will have the greatest effect when U is equal to $\frac{a}{3}$; that is, if the weight that is raised by the engine be less than the weight which would ballance the power, in the pro=

=portion of 4 to 9 and the Momentum of the weight $\frac{4Aa}{27}$."

Finding that these conclusions were far from the truth; and seeing, from many other circumstances, that the practical theory of making water and wind mills was but very imperfectly delivered by any author I had then an opportunity of consulting (a); in the year 1751 I began a course of Experi

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(a) Belidor. Architecture Hydraulique, greatly prefers the application of water to an undershot Mill, instead of an overshot; and attempts to demonstrate, that water applied undershot will do six times more execution than the same applied overshot see. Voll. I Page 286. While Desaguliers endeavouring to invalidate what had been advanced by Belidor, and greatly preferring an overshot to an undershot, says Annotat. on lecture 12th voll. II Page 532. that from his own experience "a well made overshot Mill ground as much corn in the same time with ten times less water:" so that between Belidor and Desaguliers here is a difference of no less than 60 to 1.

Again Belidor. voll II. Page 72 says, that the Centre of gravity of each sail of a wind mill should travail with one third of the velocity of the wind in its own Circle; so that taking the distance of this Centre of gravity from the Centre of Mo=

ments upon this subject. These experiments with the conclusions drawn from them, have already been communicated to this Society, who printed them in voll LI. of their Transactions for the year 1759 and for this communication I had the honour of receiving the annual Medal of Sir Godfrey Copley from the hands of our very worthy President the late Earl of Macclesfield, Those Experiments and Conclusions stand un=controverted, so far as I know to this Day, and having since that time been concerned in directing the construction of a great number of Mills, which were all executed upon the principles deduced from them, I have by that means had

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tion at 20 feet, as he states at Page 38, art. 849 the circumference will be exceeding 120 feet. English measure: a wind therefore to make the Mill go twenty turns per Minute, which they frequently do with a fresh wind and all their cloth spread, would require the wind to move above eighty miles an hour; a velocity perhaps hardly equaled in the greatest storms we experience in this Climate.

many opportunitys of comparing the effects actualy produced with the effects which might be expected from the calculation; and the agreement I have always found between these two, appears to me fully to establish the principles upon which they were constructed, when applied to great works, as well as upon a smaller scale in Models.

Respecting the explanatory deduction of Desaguliers in the first example above mentioned, which, indeed I have found to be the commonly received Doctrine among theoreti-cal Mechanics, it is shewn, Phil. Trans. Voll LI. Page 120, 121 and 123. Part I. Maxim 4. that where the velocity of water is double, the adjutage of aperture of the sluice remain-ing the same; the effect is eight times; that is, not as the square but as the Cube of the velocity: and the same is investi-gated concerning the ~~the~~ power of the Wind arising from dif-ference of velocity. Page 156 being Part 3 Maxim. 4.

The Conclusion in the second example abovementioned, adopted both by Desaguliers and Maclaurin, is not less wide of the truth than the foregoing; for if that conclusion were true, only $\frac{4}{27}$ths of the water expended could be raised back again to the height of the reservoir from which it had decended, exclusively of all kinds of friction, &

Which would make the actual quantity raised back again still less; that is, less than one seventh of the whole; where-as it appears from the table I Page 115. of the said voll. that in some of the experiments there related, even upon the small scale on which they were tried, the work done was equivalent to the raising back again about one quarter of the water expended; and in large works the effect is still greater, approaching towards half, which seems to be the limits for the undershot Mills, as the whole would be the limit for the overshot Mills, if it were possible to set aside all friction, resistance from the air &c. See Page 130.

The velocity also of the Wheel which, according to M. Parent's determination, adopted by Desaguliers and Maclaurin, ought to be no more than one third of that of the water, varies at the maximum in the above mentioned experiments of of table first, between one third and one half; but in all the cases there related, in which the most work is performed in proportion to the water expended, and which approach the nearest to the circumstances of Great works, when properly executed, the Maximum lies much nearer to one half than one third; one half seeming to be the true Maximum, if nothing were lost by the resistance of the air, the scattering of the water carried up by the Wheel, and

thrown of by the Centrifugal force, &c. all which tend to diminish the effect more, at what would be the Maximum if these did not take place, than they do when the motion is a little slower.

Finding these matters as well as others to come out in the experiments, so very different from the opinions and calculations of authors of the first reputation, who reasoning according to the Newtonian definition, must have been led into these errors from a want of attending to the proper collateral circumstances; I thought it very material especially for the practical artist, that he should make use of a kind of reasoning in which he should not be so liable to mistakes; inorder, therefore, to make this matter perfectly clear to myself and others I resolved to try a set of experiments from whence it might be inferred what proportion or quantity of mechanical power is expended in giving the same body different degrees of velocity, This Scheme was put in execution in the year 1759 and the experiments were then shewn to several friends, particularly my very worthy and ingenious friend Mr. Wm. Rufsell.

In my experimental inquiry concerning the power of water and wind before refered to, I have Page 105.

Part first, defined what I meant by power, as applied to prac=tical mechannicks, that is, what I now call mechannical power; which, in terms synonymous to those there used may be said to be measured by ~~those there mentioned~~ multiplying the weight of the Body by the perpendicular height from which it can decend; thus, the same weight, decending from a double height, is capa=ble of producing a double mechannical effect, and is therfore a double mechannical power. A double weight decending from the same height, is also a double power, because it is likewise capable of producing a double effect; and a given body, decending through a given perpendicular height, is the same ~~power as a~~ double body decending through half that perpendicular; for, by the in=tervention of proper levers, they will counterballance each other conformably to the known laws of mechannics, which have never been controverted. It must, however, be always understood, that the decending body, when acting as a measure of power, is supposed to decend slowly, like the weight of a Clock or Jack; for if quickly decending it is sensibly compounded with a=nother law, viz. the law of acceleration by gravity.

Description of the Machine

AB is the base of the Machine placed upon a table.

AC is a Pillar or standard.

CD is an arm, upon the extremity of which is fixed a plate f.g. which is here seen edge ways, through which is a small hole for receiving a steel pivot, e, fixed on the top of the upright axis eB; the lower end of this axis finishes in a conical steel point, which rests upon a small cup of hard steel polished at B. HI is a Cylinder of whole fir, which passes through a perforation in the axis, and therein fixed; and, upon the two arms formed thereby, are capable of sliding.

KL two cylindric weights of lead of equal size, which are capable of being fixed upon any part of the Cylindric arms, from ~~the axis to~~ the extremitys by means of two thin wedges of wood.

The two weights, therefore, being at equal distances from the Centre, and the axis perpendicular, the whole will be ballanced upon the point at B, and moveable thereupon by an impelling power, with very little friction.

Upon the upper part of the axis are formed M,N two Cylindrical Barrels, whereof M, is double of the Diameter of N; they have a little pin stuck into one side of each at o, p.

Q is a piece capable of sliding higher or lower as occasion requires; and carries R a light puley of about three inches diameter, hung upon a steel axis, and moveable upon

two

small pivots. The plane of the pulley, however, is not directed to the middle of the upright axis, but a little on one side so as to point (at a mean) between the surface of the bigger barrel and the less. S, is a light scale for receiving weights, and hangs by a small twine, cord, or line, that passes the pulley and terminates either upon the bigger barrel or less, as may be required, the sliding piece Q being moved higher or lower for each, that the line, in passing from the pulley to the barrel may be nearly horizontal. The end of the line, that is farthest from the scale is terminated by a small loop, which hangs ~~on~~ upon the pin a, or the pin p. according as the bigger or the lesser Barrel is to be used.

Now, having wound up a certain number of turns of the line upon the Barrel, and having placed a weight in the Scale S, it is obvious, that it will cause the axis to turn round, and give motion to its arms, and to the weights of lead placed thereon, which are the heavy Bodies to be put in motion by the impulse of the weight in the Scale; and when the line is wound off to the pin the loop slips off, and the scale then falling down, the weight will cease to accelerate the motion of the heavy Bodies, and leave them revolving equably forward, with the velocity they have acquired, except so far as it must be gradualy lessened by the friction of the Machine, and resistance of the air.

which

which being small, the bodies will revolve some time before there velocity is apparently diminished.

Measures of some Parts of the Machine

	inches
Diameter of the Cylinders of lead, or of the heavy Bodies ----	5.57
Length of D.o ----------	1.56
Diameter of the hole therein ------	.72

Weight of each Cylinder 3 Lib Averdupoise

Greater distance of the middle of each Body from the Centre of the axis. ---------	8.25
The smaller distance of D.o -------	3.92
Ten turns of the smaller Barrel raises the scale Five D.o of the bigger D.o -----	25.25

When the Bodies are at the smaller Distance above specified from the axis of Rotation, they are then in effect at half the greater distance from that axis: for, since the axis itself, and the Cylindric arms of wood, keep an unvaried distance from the centre of Rotation, the Bodies themselves must be moved nearer than half their former distance, in order that, compounded with the unvariable parts, they may be virtualy at half the distance. In order to find this half Distance nearly I put in an arm of the same wood that only went through the

the axis without extending in the opposite direction; one of the bodies being put upon the end of this arm, at the distance 8.25 inches, the whole Machine was inclined till the body and arm became a kind of Pendulum, and vibrated at the rate of 92 times per minute. and as a Pendulum of that length vibrates quicker in the proportion of $\sqrt{1}$ to $\sqrt{2}$. that is in the proportion of 92 to 130 nearly, therefore keeping the same inclination of the Machine, the weight was moved upon the arm till it made 130 vibrations per Minute; which was found to be when it was at 3.92 inches from the Centre as above stated, which is about $\frac{2}{10}$ths of an inch nearer than the half Distance the Double arm was then put in and marked accordingly, and the bodies being mounted thereon the whole was adjusted ready for use: and with it were tried the following experiments, each of which was repeated so many times as to be fully Satisfactory.

No.	Ounces Avoirdupoise in the Scale	Barrel used M the bigger N the Smaller	The arms W the whole H the half Length	No. of turns of the line wound on the Barrel	Time of the Decent of the Weight in the Scale	Time in making 20 Revolutions with equable motion
1	8	M	W	5	$14''\frac{1}{4}$	$29''$
2	8	N	W	10	$28\frac{1}{4}$	$29\frac{1}{4}$
3	8	N	W	$2\frac{1}{2}$	$14\frac{1}{4}$	$58\frac{1}{2}$
4	32	M	W	5	7	14
5	32	N	W	10	14	$14\frac{3}{4}$
6	32	N	W	$2\frac{1}{2}$	7	$28\frac{3}{4}$
7	8	M	H	5	7	$14\frac{3}{4}$
8	8	N	H	10	14	15
9	8	N	H	$2\frac{1}{2}$	7	$30\frac{1}{4}$
1	2	3	4	5	6	7

The $58''\frac{1}{2}$, in number 3, collumn. 7 was determined in fact from $29''\frac{1}{4}$, being the time of making 10 equable Revolutions after the weight was dropped off, in order to prevent the sensible retardation that might take place, and affect the observation, if continued for 20 Revolutions made so slowly.

Further Definitions

I have already defined what I mean by mechanic power; but, before I proceed, further it will be necessary to also to Define the following terms

Impulse or Impulsion,
Impulsive force or Power,
Impelling force or Power.

By all which I understand the uniform endeavour that one body exerts upon another, in order to make it move, and that ~~whe~~ whether it produces or generates motion by this endeavour or not; and the quantity of this impelling may be measured either by its being a weight of itself, or by being counterballanced by a weight. It may also act either immediately upon the Body to be moved, so that if motion is the consequence, they may move with the same velocity; and that, either by a simple contact, or by being drawn as by a cord, or pushed as by a staff: or it may act by the intervention of a leaver or other mechannic instrument, in which the velocity of the body to be moved may be very different, from the velocity of the impelling

Power

Power or mover; but in comparing them the impelling powers must be reduced according to the proportional velocities of the mover and the moved or in levers of Different lengths, they may be compared by a standard length of Lever which is the method taken in the subsequent Reasoning upon the experiments. An impelling Power, therefore, consisting of a double weight, or requiring a Double weight to counterballance it, when acting with equal leavers, is a Double impelling power, or an impelling Power of Double the intensity.

Observations and Deductions from the Preceding Experiments

1st. By the first experiment it appears, that the mechannic employed, consisting of 8 Ounces in the Scale Delibrate by decending (by 5 turns of the bigger Barrel, through a perpendicular space $25\frac{1}{4}$ inches, will represent the quantity of mechannic Power, which causes the two heavy bodies from a state of Rest, to a velocity such as to carrie them equably through 20 circumferences of their Circle of Revolution in the space of 29″ and that the time in which the Mechannic Power produced this effect $14''\frac{1}{4}$, as appears by column 6th and this mechannic power we shall express by the number 202, the product of ~~the number~~ of

of the number of Ounces in the Scale, multiplied by the inches in its perpendicular Decent. for $8 \times 25\frac{1}{4} = 202$

2^d. By the second experiment, as 10 turns of the lesser Barrel are equal to the same perpendicular height as 5 turns of the Bigger, it follows that the same mechanic Power viz. 202. acting upon the same heavy Bodies, to accelerate them, produces the very same effect in generating motion in the Bodies as before viz 20 Revolutions in 29‴ 4 the small difference of $\frac{1}{4}$ of a second being no more than may be reasonably attributed to the unavoidable errors, arising from friction of the machine, want of perfect accuracy in its measures resistance of the air, and imperfections in the observations themselves, which must not only be allowed for in this, but the rest; but as the impelling power is acting here upon a leaver of but half the length, and consequently but half the intensity when referred to the bodies to be moved, it takes just double the time to generate the same velocity therein.

Deduction. It appears from hence that the same mechanic Power is capable of producing the same velocity in a given body, whether its applied so as to produce it in a greater or a lesser time; but that the time taken to produce a given velocity, by an uniform continued action,

is in a simple inverse proportion of the intensity of the impelling Power.

3.^{dly} The experiment being made with 2½ turns of the lesser Barrel, the same weight of 8 Ounces decending in the scale only one quarter part of the former perpendicular, the mechannic power employed will only be one quarter part of the former, Viz. 50½; but as one quarter part of the mechannic Power produces one half of the former velocity in the heavy Bodies; that is, they make 20 in 58.″½ that is, nearly 10 Revolutions in 29;″ we may conclude in this instance that the mechannic Power, employed in producing motion, is as the square of the velocity produced in the same body; and that the velocity produced is as the time that an impelling Power, of the of the same intensity, continues to act upon it, as appears by the near agreement of Numbers 2 and 3 collumn. 6.th

4.^{thly} In the fourth experiment, the aparatus is the same as the first, only here the weight in the scale is 32 ounces; that is, the impelling power is the quadruple of the first and hereby a double velocity is given to the Bodies, for they make 20 Revolutions in 14″ which is a small matter less, than half the time taken up in making 20 Revolutions in the first experiment. It also appears that the velocity acquired is simply as the impelling Power compounded with the time of its action, for a quadruple impulsion acting for 7″ instead of 14 generates a double velocity, while the mechan

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Power employed to generate it is quadruple, for $32 \times 25\frac{1}{4} = 808$.
And here the mechannic power employed being four times greater than the first, it holds here also, that the mechannic power to be necessarily employed is as the square of the velocity to be generated; that is, in the same proportion as turned out in the third experiment where the mechannic power employed was only a quarter part of the first.

5thly. The fifth and sixth experements were made with a mechannic, power four times greater than those employed in numbers 2 and 3 respectively; and since the same deductions result from hence as from numbers 2 and three, they are additional conformations of the conclusions drawn from them and from the ~~fourth~~ last article.

6thly. In the seventh experiment, the disposition of the apparatus is the same as number 1, only here the Bodies are placed upon the arms at half length; from which it appears that the same mechannic power still produces the same velocity in the same bodies; for though 20 Revolutions were performed in $14\frac{3}{4}''$ (see coll. 7) which is nearly half the time that 20 Revolutions were performed in the first experiment, yet since the circles in which the bodies revolved in the seventh are only of half the Circumference as those of number 1, it is obvious, that the absolute velocity aquired by the moving in the two Cases is equal. But by column 6th the time in which it was generated is only half; yet notwithstanding,

this

this will coincide with the former conclusions, if the intensity of the impelling power is compounded therewith; for though the Barrel was the same with the same number of turns as in number 1, and therefore the leaver the same, by which the impelling power acted, yet as the Bodies upon which this lever was to act were placed upon a lever of only half the length from the centre; the impelling power acting upon the first lever, would act upon the second with double the intensity; according to the known laws of mechannics; that is it would require a double weight opposing the bodies, to prevent their moving in order to ballance it.

An impulsive power therefore of double the intensity, acting for half the time produces the same effect in generating motion, as an impulsive power, of half the intensity acting for the whole time

7thly The eighth and ninth experiments afford the same deductions and conformations relative to the seventh experiments that the fifth and sixth do respecting the fourth, and the second and third respecting the first; and from the near agreement of the whole, when the necessary allowances before mentioned are made; together with some small inequality arising from the mechannical power lost by the difference of the motion given by gravity to the weight in the scale: I say, from these agreements, under the very Different mechannical powers applied, which were varied in the proportion of 1 to 16.

we

we may safely conclude, that this is the universal law of nature respecting the capacities of Bodies in motion, to produce mechanical effects, and the mechanic power necessary to be employed to produce or generate different velocitys (the Bodies being supposed equal in their quantity of matter;) that the mechanic powers to be expended are as the squares of the velocitys to be generated and vice versa; and that the simple velocitys generated are as the impelling power compounded with or ~~generated~~ multiplied by the time of its action and vice versa."

We shall perhaps, form a still clearer conception of the relation between velocities produced, and the quantity of mechanic required to produce them; together with the collateral circumstances attending, by which these propositions seeming by ~~this~~, are reconciled and united, by stating the following popular elucidation, which indeed was the original idea that occurred to me on considering this subject: to put which to an experimental proof gave birth the foregoing apparatus and experiments

Suppose then a large iron Ball of 10 feet Diam. turned truly spherical, and set upon an extended plain of the same metal, and truly level. Now; if a man begins to push at it he will find it very resisting to motion at first; but by continuing the impulse he will gradually get it into motion, and having nothing to resist it but the

the air, he will by continuing his efforts at length get it to roll almost as fast as he can run, suppose now, in the first minute he gets it rolled through the space of one yard; by this motion proceeding from rest (similar to what happens to falling bodies) it would continue to roll forward at the rate of two yards per minute without further help; but supposing him to continue his endeavours, at the end of another minute he will have given it a velocity capable of carrying it through two yards more in addition to the former, that is at the rate of 4 yards per minute; and at the end of the third time, he has again added an equal increase of velocity, and made it proceed at the rate of six yards per minute; and so on, increasing its velocity at the rate of two yards in every minute. The man therefore, in the space of every minute, exerts an equal impulse upon the ball, and generates an equal increase of movement upon the ball correspondent to the definition of Sir Isaac Newton. But let us see what happens besides: the man in the first minute, has moved but one yard from where he set out, but he must in the second minute move two yards more in order to keep up with the ball, and as he has exerted an impulse upon it, so as at the end of the second minute to have given it an additional velocity of the two yards, he must also on this

time have gradualy changed its velocity from the rate of two yards per minute to that of four, and the space that he will of consequence have actually been oblidged to go through in the second minute, will be according to the mean of the extremes of velocity at the beginning and end thereof, that is three yards in the second minute, so that being one yard from his original place at the beginning of the second minute, at the end of it he will have moved the sum of the journies of the first and second minute that is, in the whole four yards from his orriginal place.

As he has now generated a velocity in the ball of 4 yards per minute, in the third he must travail 4 yards to keep up with the ball, and one more in generating the equal incremen of velocity, so that in the third minute he must travail five yards to keep up ~~with with~~ the same impelling power upon the ball that he did in the first minute in travelling one. so that these five yards in the third minute, added to the four yards that he traveled in the two preceeding minutes, sets him at the end of the third minute nine yards from where he set out, having then given the ball a velocity capable of carrying it uniformly forward, at the rate of six yards per minute as before stated.

We may now leave the further pursuit of these proportions and se how the account stands. He generated a velocity of

of two yards per minute, in the first minute the square of which is four when he had moved but one yard from his place; and had generated a velocity of six yards per minute. the square of which is thirty six. at the end of the third minute when he had travelled nine yards from his place. Now since the square of the velocity, generated at the end of the first minute, is to that of the velocity Generated at the end of the third minute. as 4 : 36, that is as 1 to 9 and since the spaces moved through by the man to communicate these velocities are also as 1 to 9, it follows, that the spaces through which the man must travel in order to generate these velocities respectively (preserving the impelling perfectly equal,) must be as the squares of the velocities that are communicated to the Ball, for if the man was to be brought back again to his origenal place by a mechannical power, equaly exerted upon the man equally resisting this would be the the measure of what the man has Done in order to give motion to the Ball. It therefore directly follows, conformably to what has been Deduced from the experiments, that the mechannic Power that must of necessity be employed in giving Different degrees of velocity to the same body must be as the square of that velocity, and if the converse of that proposition did not

not hold, Viz. that if a body in motion, in being stopped, would not produce a mechanical effect equal or proportional to the square of its velocity, or to the mechanical power employed in producing it. the effect would not correspond with its producing cause.

Thus the consequences of generating motion upon a level plane exactly correspond with the generating of motion by gravity; Viz. that though in two seconds of time the equal impulsive power of gravity gives twice the velocity to a body that it does in one second, yet this collateral circumstance attends it, that at the end of the double time, in Consequence of the velocity acquired in the first half, the Body has fallen from where it set forward through four times the perpendicular; and therefore, though the velocity is only Doubled, yet four times the mechanical power must be expended in bringing up the fallen body to its first place.

This then appears to be the foundation, not only of the disputes that have arisen, but of the mistakes that have been made, in the application of the different definitions of quantity of motion, that while those, that have adhered to the definition of Sir Isaac Newton have complained of their adversaries, in not considering the time in which the effects are produced, they themselves have not always taken into the account the space that the impelling power is obliged to travel through in producing the different degrees of velocity.

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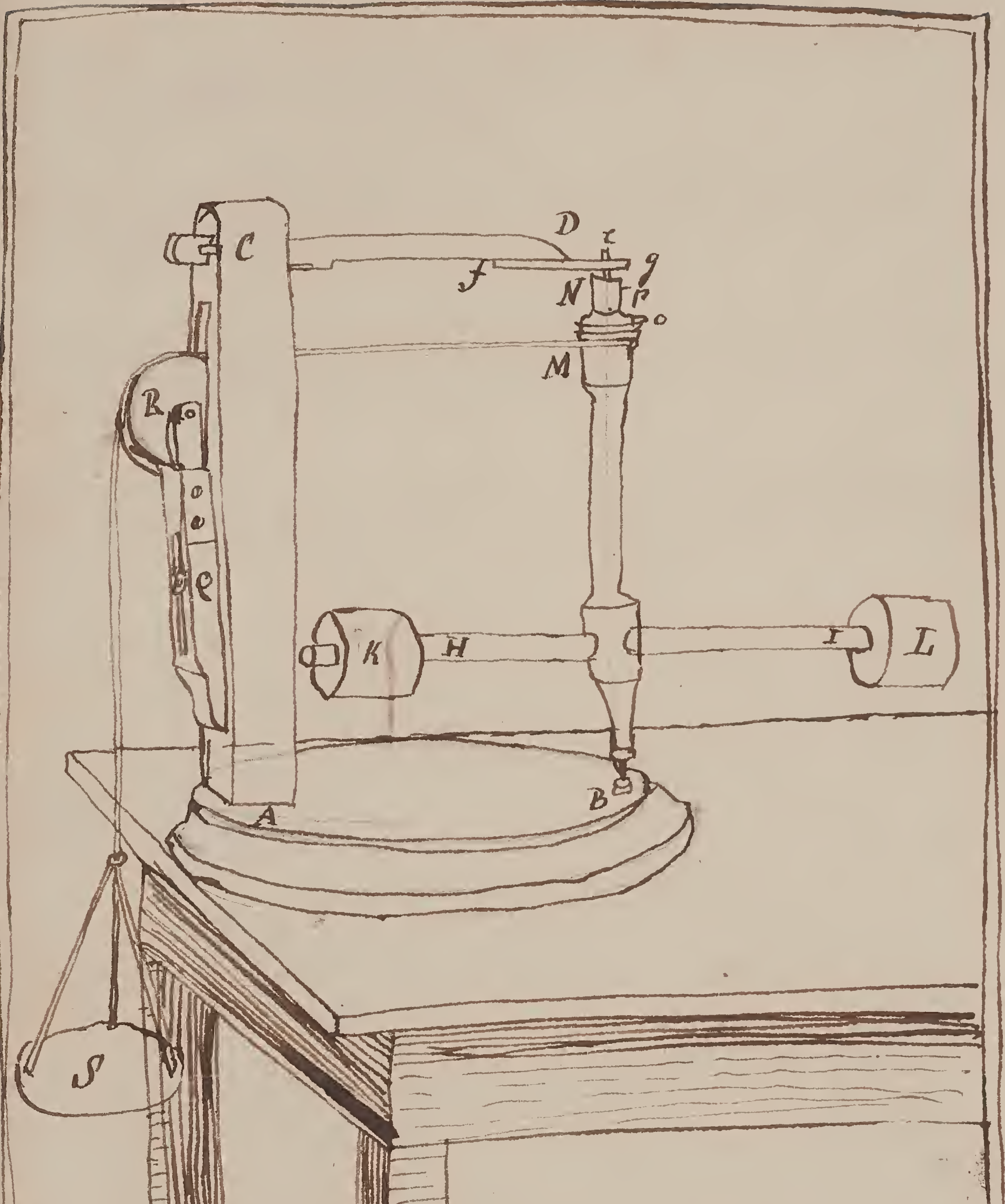
seems therefore, that, without taking in the collateral circum-stances both of time and space, the terms, quantity of motion or Momentum, and force of Bodies in motion, are absolutely indefinite: and that they cannot be so easily, distinctly, and fundamentally compared, as by having recourse to common measure viz. mechanic Power.

From the whole of what has been investigated, it there-fore appears, that time properly speaking, has nothing to do with the production of mechanical effects, otherwise than as by equally flowing, it becomes a common measure; so that whatever mechan-nical effect is found to be produced in a given time, the uniform continuance of the action of the same mechanical power will, in a double time, produce two such effects, or twice that effect. A Mechanical power, therefore, properly speak-ing, is measured by the whole of its mechanical effects produc-ed whether that effect is produced in a greater or a lesser time: thus; having treasured up 1000 tuns of water which I can let out upon the overshot wheel of a mill and descending through a perpendicular of 20 feet, this power applied to proper mechan-nic instruments, will produce a certain effect that is it will grind a certain quantity of corn, and that at a certain rate of expending it, it will grind this corn in an hour. But suppose

the

the mill equally addapted to produce a proportionable effect, by the application of a greater impulsive power as with a less, then if I let out the water twice as fast upon the wheel, it will grind the corn twice as fast, and both the water will be expended and the corn ground in half an hour. Here the same mechannical effect is produced, Viz. the grinding a given quantity of corn by the same mechannical power, Viz. 1000 tuns of water decending through a given perpendicular of 20 feet, and yet this effect is produced in one case in half the time of the other. What time therefore has to do in the business is this, let the rate of doing the business, or producing the effect, be what it will, If this rate is uniform, when I have found by experiment what is done in a given time, then, proceeding at the same rate twice the effect will be produced in twice the time, on supposition that I have a supply of mechannic power to go on with. Thus 1000 tuns of water, decending through 20 feet of perpendicular, being, as has been shewn, a given mechannic power, let me expend it at what rate I will, if when this is expended, I must wait another hour be=fore it be renewed by the natural flow of a river, or otherwise, I can then only expend twelve such quantitys of powe in 24 hours; but if while I am expending 1000 Tuns in one hour, the stream renews me the same quantity, then I can expend

24 such quantities in 24 hours; that is, I can go on continually at that rate, and the product or effect, will be in proportion to time, which is the common measure; but the quantity of mechanic power arising from the flow of the two rivers, compared by taking an equal portion of time, is double in the one to the other, though each has a mill, that, when going, will grind an equal quantity of corn in an hour.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New fundamental Experiments upon the Collision of Bodies. By Mr. John Smeaton F.R.S. — 1782 —

To Sir Joseph Banks. Bart. F. R. S. —

Sir The Subjects of the inclosed tract have been the object of my consideration for many years past: and as they contain some matters that have not only been variously reasoned about, but variously concluded upon; if what is contained therein shall appear of such a nature as either to establish truth, as it appears to me or to prompt some more able person, in reviewing the subject, to shew what links in my chain of reasoning thereon are defective, so as to establish the whole doctrine of moving bodies upon one plain consistent basis, my end will be equally answered in offering them to you, to be laid before the Royal Society, in case you shall think that the importance of the subject shall merit the same: furthermore, I hope to be forgiven, if in some parts of this paper I have expressed myself with more pointedness than I might have done, for I declare, that it was solely owing to my earnestness that the subject of these mechanic motions & powers should be fully & freely investigated, and established upon grounds that shall be uncontrovertable. I have the honour to be &c —

It is universally acknowledged, that the first simple principles of Science cannot be too critically examined, in order to their being firmly established; more especially those which relate to the practical & operative parts of Mechanics, upon which much of the Active business of mankind depends. A sentiment of this kind occasioned my tract upon Mechanic Power, which was published in the Phil. Trans. vol. 66. for the year 1776. What I have now to offer was intended as a Supplement thereto, & the experiments were then, in part, tried; but the completion thereof was deferred at that time, partly from want of leisure; partly to avoid too great a length of the paper itself; and partly to avoid bringing forward so many points at once. My present purpose is to show, that the true doctrine of the collision of bodies hangs as it were upon the same hook, as the doctrine of the gradual generation of Motion from rest, considered in that paper; that is, that whether bodies are put into gradual Motion, and uniformly accelerated from rest to any given velocity; or are put in motion, in an instantaneous manner; when bodies of any kind strike one another, the motion, or sum of the motions produced, has the same relation to Mechanic Power therein defined, which is necessary to produce the Motion desired. To prove this, & at the same time to show some capital mistakes in principle, which have been assumed as indisputable truths by men of great learning, is the reason of my now pursuing the same subject.

I do not mean to point out the particular mistakes which have been made by particular men, as that would lead me into too great a length; I shall therefore content myself with observing, that the laws of collision, which have been investigated by Mathematical Philosophers, are principally of three kinds; viz those relating to bodies perfectly elastic; to bodies perfectly unelastic, and perfectly soft; & to bodies perfectly unelastic, and perfectly hard. To avoid prolixity, I shall consider in each only the case of two bodies which are equal in weight or quantity of matter striking one another. Respecting those which are perfectly elastic, it is universally agreed, that when two such bodies strike one another, no motion is lost; but that in all cases, what is lost by one is acquired by the other: & hence, that if an elastic body in motion strikes another at rest, upon the stroke the former will be reduced to a state of rest, and the latter will fly off with an equal velocity. —

In like manner, if a none elastic soft body strikes another at rest, they neither of them remain at rest, but proceed together from the point of collision with exactly one half of the velocity that the first had before the stroke; this is also universally allowed to be true, and is fully proved by every good experiment on the Subject. —

Respecting the third species of body, that is, those that are non-elastic, and yet perfectly hard; the laws of motion relating to them, as laid down by one species of Philosophers, have been rejected by another; the latter alledging, that there are no such bodies to be found in nature whereon to try the experiment; but those who have laid down & assigned the doctrine that would attend the collision of bodies of this kind (if they could be found) have universally agreed, that if a non-elastic hard body was to strike another of the same kind at rest, that, in the same manner as is agreed concerning non-elastic soft bodies, they neither of them would remain at rest, but would in like manner proceed from the point of collision, with exactly one half of the velocity that the first had before the stroke: in short, they lay it down as a rule attending all non-elastic bodies, whether hard or soft, that the velocity after the stroke will be the same in both, viz one half of the velocity of the original striking body.—

Here is therefore the assumption of a principle, which in reality is proved by no experiment, nor by any fair deduction of reason that I know of, viz that the velocity of non-elastic hard bodies after the stroke must be the same as that resulting from the stroke of non-elastic soft bodies; and the question now is whether it is true or not?

Here it may be properly asked, what ill effects can

result to practical men, if Philosophers should reason wrong concerning the effects of what does not exist in nature, since the practical men can have no such materials to work upon, or misjudge of? But it is answered, that they who infer an equality of effects between the two sorts, may from thence be misled themselves, & in consequence mislead practical men in their reasonings and conclusions concerning the sort with which they have abundant concern, viz. the non-elastic soft bodies, of which water is one, which they have much to do with in their daily practice.

Previous to my trying my experiments on Mills I never doubted the truth of the doctrine, that the same velocity resulted from the stroke of both sorts of non-elastic bodies; but the tryal of those experiments made me clearly see at least the inconclusiveness, if not the falsity of that doctrine; because I found a result which I did not expect to have arisen from either sort; and for which, when it appeared from experiment, I could see a substantial reason why it should take place in one sort, & that it was impossible it could take place in the other; for if it did, the bodies could not have been perfectly hard which would be contrary to the hypothesis. Of this deduction I have given notice on my said tract on Mills, published in the Phil. Transac., vol LI, for year 1779.*

"* The effect, therefore, of overshot wheels, under the same circumstance of quantity "and fall, is at a medium double to that of the undershot: and as a consequence "thereof, that non-elastic bodies, when acting by their impulse or collision, communicate only "a part* of their original power; the other part being spent in changing their figure "in consequence of the stroke. Phil. Trans. vol LI. p. 133. —

It may also be said, that since we have no bodies perfectly elastic, or perfectly unelastic & soft. why should we expect bodies perfectly unelastic & hard? Why may not the effects be such as should result from a supposition of their being imperfectly elastic joined with their being imperfectly hard? But here I must observe, that the supposition appears to be a contradiction in terms. –

We have bodies which are so near perfectly Elastic, that the laws may be very well deduced & confirmed by them; and the same obtains with respect to non-elastic soft Bodies; but concerning bodies of a mixed nature, which are by far the greatest number, so far as they are wanting in elasticity, they are soft, and bruise, yield, or leave a mark in collision; and so far as they are not perfectly soft they are elastic, and observe a mixture of the laws relative to each; but imperfectly elastic bodies, imperfectly hard come in reality under the same description as the former mixed bodies: for so far as they are imperfectly hard they are soft, an either bruise & yield, or leave a mark in the stroke; and so far as they want imperfect elasticity, they are non-elastic; that is to say, they are bodies imperfectly elastic, and imperfectly soft; and in fact I have never yet seen any bodies but what came under this description. It seems therefore, that respecting the hardness of bodies they differ in degrees of it, in proportion as they have a greater degree of tenacity or cohesion; that is, are further removed from perfect softness, at the same time that their elastic springs, so far as they reach, are very stiff; and hence we may (by the way) conclude, that

the same Mechanic power that is required to change the figure in a Small degree of those bodies that have the popular appellation of hard bodies, would change it in a great degree in those bodies that approach towards softness, by having a small degree of tenacity or cohesion. In the former kind we may rank the hardest kinds of cast Iron, and in the latter soft tempered clay

While the Philosophical world was divided by the dispute about the Old and new opinion, as it was called, concerning the powers of bodies in Motion, in proportion to their different velocities: those who held the Old opinion contending, that it was as the velocity simply, asked those of the new, How, upon their principles, they would get rid of the conclusions arising from the doctrine of unelastic perfectly hard bodies? They replied, They found no such bodies in nature, and therefore did not concern themselves about them. On the other hand, those of the new opinion asked those of the Old, How they would account for the case of non-elastic soft bodies, where, according to them, the whole Motion lost by the striking body was retained in the two after the stroke (the two bodies moving together with half the velocity), though the two non-elastic bodies had been bruised & changed their figure by the stroke; for, if no Motion was lost, the change of the figure must be an effect without a cause? To obviate this, those of the Old opinion seriously set about proving, that the bodies might change their figure, without any loss of Motion in either of the striking bodies. —

Neither of these answers have appeared to me satisfactory, especially since my Mill experiments: for with respect to the first, it is no proper Argument to urge the impossibility of find-
-ing

-ing the proper Material for an experiment, in answer to a con-
-clusion drawn from an Abstract Idea. On the other hand, if it can be shewn, that the figure of a body can be changed, without a power, then, by the same law, we might be able to make a forge hammer work upon a mass of soft Iron, without any other power than that necessary to overcome the friction, resistance, and ori-
-ginal vis inertiæ, of the parts of the machine to be put in motion; for, as no progressive motion is given the mass of Iron by the hammer (it being supported by the Anvil), no power can be ex-
-pended that way; and if not is lost to the hammer by the changing the figure of the Iron, which is the only effect pro-
-duced, then the whole power must reside in the hammer, and it would jump back again to the place from whence it fell, just in the same manner as if it fell on a body perfectly elastic, upon which, if it did fall, the case would really happen: the power, therefore, to work the hammer would be the same whether it fell upon an elastic or non-elastic body; an Idea so very contrary to all experience, and over apprehension of both the Philosopher & Vulgar Artist, that I shall here leave it to its own condemnation. —

As nothing, however, is so convincing to the mind as expe-
-riments obvious to the senses, I was very desirous of contriving an experiment in point; and as I saw no hopes of finding matter to make a direct experiment, I turned my mind to an indirect one; so circumscribed, however, as to prove incontestibly, that the result of the stroke of two non-elastic perfectly hard bodies could not be the same as would result from the collision of two soft ones; that is

if it can be bona fide proved, that one half of the original power is lost in the stroke of soft bodies by the change of figure (as was very strongly suggested by the mill experiments); then since no such loss can happen in the collision of bodies perfectly hard, the result & consequence of such a stroke must be different.

The consequence of the stroke of a body perfectly hard, but void of elasticity, must doubtless be different from that of bodies perfectly elastic: for having no spring the body at rest could not be driven off with the velocity of the striking body, for that is the consequence of the action of the spring or elastic parts between them, as will be shewn in the result of the experiments; the striking body will therefore not be stopped, and as the motion it loses must be communicated to the other, from the equality of action & reaction, they will both proceed together, with an equal velocity, as in the case of non-elastic soft bodies: the question, therefore, that remains is, what that velocity must be? — It must be greater than that of the non-elastic soft bodies, because there is no mechanical power lost in the stroke. It must be less than that of the striking body, because, if equal, instead of a loss of motion by the collision, it will be doubled. If, therefore, non-elastic soft bodies lose half their motion, or mechanical power, by change of figure in collision, and yet proceed together with half the velocity, and the non-elastic hard bodies can lose none in any manner whatever; then, as they must move together, their velocities must be such as to preserve the equality of mechanic power, unimpaired, after the

the stroke the same as it was before. --

For example, let the velocity of the striking body before the stroke be 20, and its mass or quantity of matter 8; then according to the rule deduced from the experiments in the tract on Mechanic Power (see exp. third & fourth) that power will be expressed by 20 X 20 = 400, which X 8 = 3200; and if half of it is lost in the stroke, in the case of non-elastic soft bodies, it will be reduced to 1600; which divided by 16 the double quantity of matter, will give 100 for the square of their velocity; the square root of which being 10, will be the velocity of the two non elastic soft bodies after the stroke; being just one half of the original velocity, as it is constantly found to be. But in the nonelastic hard bodies, no power being lost in the stroke, the Mechanic Power will remain after it, as before it = 3200; this, in like manner, being divided by 16, the double quantity of matter, will give 200 for the square of the velocity, the square root of which is 14.14, &c. for their velocity after the stroke, which is to 10, the velocity of the non-elastic soft bodies after the stroke, as the square root of 2 to 1; or as the diagonal of a square to its side. --

It remains, therefore, now to be proved, that precisely half of the Mechanic power is lost in the collision of non-elastic soft bodies; for which purpose my mind suggested the following reflections. In the collision of elastic bodies the effect seemingly instantaneous, is yet performed in time; during which time the natural springs residing in elastic bodies, and which constitute them such, are bent or forced, till the

motion of the striking body is divided between itself & the body at rest; and in this state the two bodies would then proceed together, as in the case of non-elastic soft bodies; but as the Springs will immediately restore themselves in an equal time, and with the same degree of _impulsive force_, where-with they were bent in this reaction, the motion that remained in the striking body will be totally destroyed, and the total exertion of the two Springs, communicated to the original resting body, will cause it to fly off with the same velocity wherewith it was struck. –

Upon this Idea, if we could construct a couple of bodies in such a way that they should either act as bodies perfectly elastic; or that their springs at pleasure should be hooked up, retained or prevented from restoring themselves, when at their extreme degree of bending; and if the bodies under these circumstances observed the laws of collision of non-elastic soft bodies, then it would be proved, that one half of the mechanical power, residing in the striking body, would be lost in the Action of collision; because the impulsive force or power of the Spring in its restitution being cut off, or suspended from Acting, which is equal to the im-pulsive force or power to bend it (and which alone has been employed to communicate motion from one body to the other), it would make it evident, that one half of the impulsive force is lost in the Action, as the other half remains _locked up_ in the Springs. It also follows, as a _collateral circumstance_, that be the impulsive power of the Springs what it may from first to last, yet as one half of the _time_ of the Action is by

this means cut off; in this sense also it will follow, that one half of the Mechanic Power is destroyed; or rather, in this case, remains locked up in the Springs, capable of being <u>re-exerted</u> wherever they are set at liberty, and of producing a fresh Mechanical effect, equivalent to the motion or Mechanical power of the two non-elastic soft bodies after their collision. —

Hence we must infer, that the quantity of Mechanical power expended in displacing the parts of non-elastic soft bodies in collision, is exactly the same as that expended in bending the Springs of perfectly elastic bodies; but the difference in the ultimate effect is, that in the non-elastic soft bodies, the power taken to displace the parts will be totally lost & destroyed, as it would require an equal Mechanic power to be raised afresh, and exerted in a contrary direction to restore the parts back again ~~to their former~~ to their former places; whereas in the case of elastic bodies, the operation of half the Mechanic power is, as observed already, only locked up & suspended, and capable of being re-exerted without a further original Accession. —

These Ideas arose from the result of the experiments tried upon the Machine described in my said tract on Mechanic power, and were also communicated to my very worthy and ingenious friend Willm Russel Esqr F.R.S. at the same time that I shewed him those experiments in 1759; but the mode of putting this matter to a full and fair Mechanical trial has since occurred; and though some

rough trials, sufficient to shew the effects were made thereon, prior to the offering the paper on Mechanical Power to the Society in the 1776, yet the machine itself I had not leisure to compleat to my satisfaction till lately: which I mention to apologize for the length of time that these speculations have taken in bringing forward. —

Description of the machine for collision

Fig. 1 shews the front of the machine as it appears at rest when fitted for use. —

A is the pedestal, and AB the pillar, which supportes the whole, C, D are two compound bodies of about a pound weight each, but as nearly equal in weight as may be. These bodies are alike in construction, which will be more particularly explained by fig. 2. These bodies are suspended by two white fir rods of about half an inch diar. of ? fg.h, being about four feet long from the point of suspension to the center of the bodies; and their suspension is upon the cross-piece II, which is mortoised through, to let the rods pass with perfect freedom; and they hang upon two small plates filed to an edge on the underside, and pass through the upper parts of the rods. Their centers are at k and l, and the edges being let into a little notches, on each side the mortoise, the rods are at liberty to vibrate freely upon their respective points (or rather edges) of suspension, and are determined to one plane of vibration. MN is a flat arch of white wood which may be covered with paper, that the marks thereupon may be the more conspicuous

The cross piece II is made to project so far before the pillar, that the bodies in their vibrations may pass clear of it, without danger of striking it; and also the Arch MN is brought so far forward as to leave no more than a clearance, sufficient for the rods to vibrate freely without touching it..

Fig 2. Shews one of the compound bodies, drawn of it full size. AB is a block of wood, and about as much in breadth as it is represented in height, through a hole in which the wood rod CC passes, and is fixed therein.

DB represents a plate of lead about three-eights of an inch thick, one on each side, screwed on by way of giving it a competent weight. dBefg represents the edge of a springing plate of brass, rendered elastic by hard hammering it is about 5/8 of an inch in breadth, and about 1/20 of an inch thick. It is fixed down upon the wooden block at its end dB by means of a bridge plate, whose end is shown hi; and is screwed down on each side the spring plate by a screw which being relaxed the spring can be taken out at pleasure, and adjusted to its proper situation. kl is a light thin slip of a plate, whose under edge is cut into teeth like a fine saw or ratchet, and is attached to the spring by a pin at k, which passes through it, and also through a small stud rivetted into the back part of the spring and upon which pin, as a center, it is freely moveable.—

m.n shews a small plate or stud seen edgeways raised

upon the bridge plate, through an hole in which Stud the ratchet passes; and the lower part of the hole is cut to a tooth shaped properly to catch the teeth of the ratchet, and retain it together with the Spring to any degree to which it may be suddenly bent; and for this intent it is kept bearing gently downward, by means of a wire Spring opq, which is in reality double, the bearing part at o being semicircular; from which branching off on each side the rod cc, passes to p, and fixes at each end into the wood at q. However, to clear the ratchet which is necessarily in the middle as well as the rod, the latter is perforated; and also the block is cut away, so far as to set the main Spring at e free of all Obstacles that would prevent its play from the point B. The part fg is shown thicker than the rest, by being covered with thin kid leather tight sewed on, to prevent a certain jarring that otherwise takes place on the meeting of the Springs in collision. —

Let us now return to Fig. 1. the marks upon the arch MN are put on as follows. op is an arch of a circle from the center l, and qr an arch of a circle from the center k intersecting each other at s. Now the middle line of the marks t, v, are at the same distance from the middle line at s that the centers k l are; so that when each body hangs in its own free position, without bearing against the other, the rod ef will cover the mark at t, and the rod gh will cover the mark at v. From the point s upon the arches sp and sq respectively, set off points at an equal & competent distance from s each way, which will

give the middle of the mark w and x: and upon the arch Sp find a middle point between the mark v and w, which let be y; and on the other side, in like manner, upon the arch Sq find a middle point for the mark z; then set off the distance Sv or St from y each way, and from z each way; and from these points, drawing lines to the respective centers l and k, they will give the place and position of the marks a, b, and c, d; and thus is the machine prepared for use. —

For Trials on Elastic Bodies

For this use take out the pins and ratchets from each respectively, and the springs being then at liberty, with a short bit of stick (suppose the same size as the rods) turn aside the rod gh with the right hand, carrying the body D upwards till the stick is on the mark w, as suppose at ⊙; there hold it, and with the left set the body C perfectly at rest; in which case the rod ef will be over the mark t; then suddenly withdraw the stick, in the direction that the rod gh is to follow it, and the spring of the body D, impinging upon that of the body C, they will be both bent, and also restored; and the body C will fly off, and mount till its rod ef covers the mark x; the rod of the striking body D remaining at rest upon its proper mark of rest v, till the body C returns, when the body D will fly off in the same manner; the two bodies thus rebounding a number of times, losing a part of their vibration each time; but so nearly is the

theory of elastic bodies fulfilled hereby, that the single advantage of originally pushing the rod gh beyond the mark w, by the thickness of the stick, or its own thickness, is sufficient to carry the rod of the quescent body C compleatly to its mark x.

There are several other experiments that may be made with this apparatus, in confirmation of the doctrine of the collisions of Elastic bodies; which being universally agreed upon, and well known, it is needless further to dwell upon here; but respecting the application to non-elastic soft bodies, it is far more difficult to come at a fitness of materials for this kind of experiments, than it is for those supposing perfect Elasticity. The conclusions however, may be attained with equal certainty.

For trials on non-elastic Soft Bodies

For this purpose the ratchets must be applied and put in order as before described, and the Springs being both put to their point of rest, let the body D be put to its mark w in the same manner as before described, and the body C to rest. The body D being let go, and striking the body C at rest, in consequence of the stroke, the Springs being hooked up by the ratchets, they both move from their resting marks t, v, respectively toward M: Now if they both moved together, and the rod ef covered the mark c, and the rod gh covered the mark d at their utmost limit, then they would truly

obey the laws of non-elastic soft bodies; because their medium ascent would be to the mark z, which is just half the angle of ascent to the mark x; but as in this piece of machinery, though the main or prin-ciple springs are hooked up, yet every part of them, and all the materials of which they are composed, and to which they are attached, have a degree, or more properly speaking, a certain compass of elasticity, which, as such, is perfect, and no motion lost thereby.

We must not, therefore, expect the two compound bodies after the stroke to stick together without seperating, as would be the case with bodies truly non-elastic & soft; but that from the elasticity they are possessed of, they will by rebounding be seperated; but that elasticity being perfect, can occasion no loss of motion to the sum of the two bodies; so that if the body C ascends as much above its mark c as the body D falls short of its mark d, then it will follow, that their medium ascent will still be to the mark z, as it ought to have been, had they been truly non-elastic soft bodies; and this, in reality, is truly the case in the experiment, as nearly as it can be discerned. –

After a few vibrations, by the rubbing of the springs against one another, they are soon brought to rest; and here they would always rest had they been truly and properly perfect non-elastic soft bodies; but here, as in the case of these bodies, by a change of the figure and situa-tion of the component partes, there is expended one half of the mechanical power of the first mover, yet in

this case the other half is not _lost_, but _suspended_, ready to be re-exerted whenever it is set at liberty; and that it is really of _bona fide one half_ and neither more or less, appears from this uncontroverted simple principle, that the power of restitution of a perfect Spring is exactly equal to the power that bends it. and this may, in a certain degree be shewen to be fact by experiment, if there were any need of such a proof; for if, when the bodies are at rest after the last experiment, the two rods are lashed together near the bottom with a bit of thread, and then the ratchets unpinned and removed; on cutting the thread with a pair of scissars they will each of them rebound C towards M, and D towards N; and if they rebounded respec-tively to z and y, the mechanical power exerted would be the same as it was after the stroke, when the mean of their two ascents was up to the mark z; but here it is not to be expected, because not only the motion lost by the friction of the ratchets is to be deducted, because it had the effect of real non-elasticity; but also the elasticity that seperated them in the stroke, which was lost in the vibrations that succeeded; neither of which hindered the mean ascent to be to z; but yet, under all these disadvantages in the machine (if not unreasonably ill made) the rod ef will ascend to d, and gh to a; and hence I infer, as a positive truth, that in the collision of non-elastic soft bodies, one half of the mechanic power residing in the striking body is lost in the Stroke

Respecting bodies unelastic and perfectly hard, we must

infer. that since we are unavoidably led to a conclusion concerning them, which contradicts what is esteemed a truth capable of the strictest demonstration; viz that the velocity of the Center of gravity of no System of bodies can be changed by any collision betwixt one another, something must be assumed that involves contradiction. This perfectly holds, according to all the established rules, both of perfectly elastic and perfectly non-elastic soft bodies; rules which must fail in the perfectly non-elastic hard bodies, if their velocity ~~if their velocity~~ after the stroke is to the velocity of the striking body as one is to the square root of 2; for then the center of gravity of the two bodies will by the stroke acquire a velocity greater than the center of Gravity the two bodies had before the stroke in that proportion, which is proved thus.—

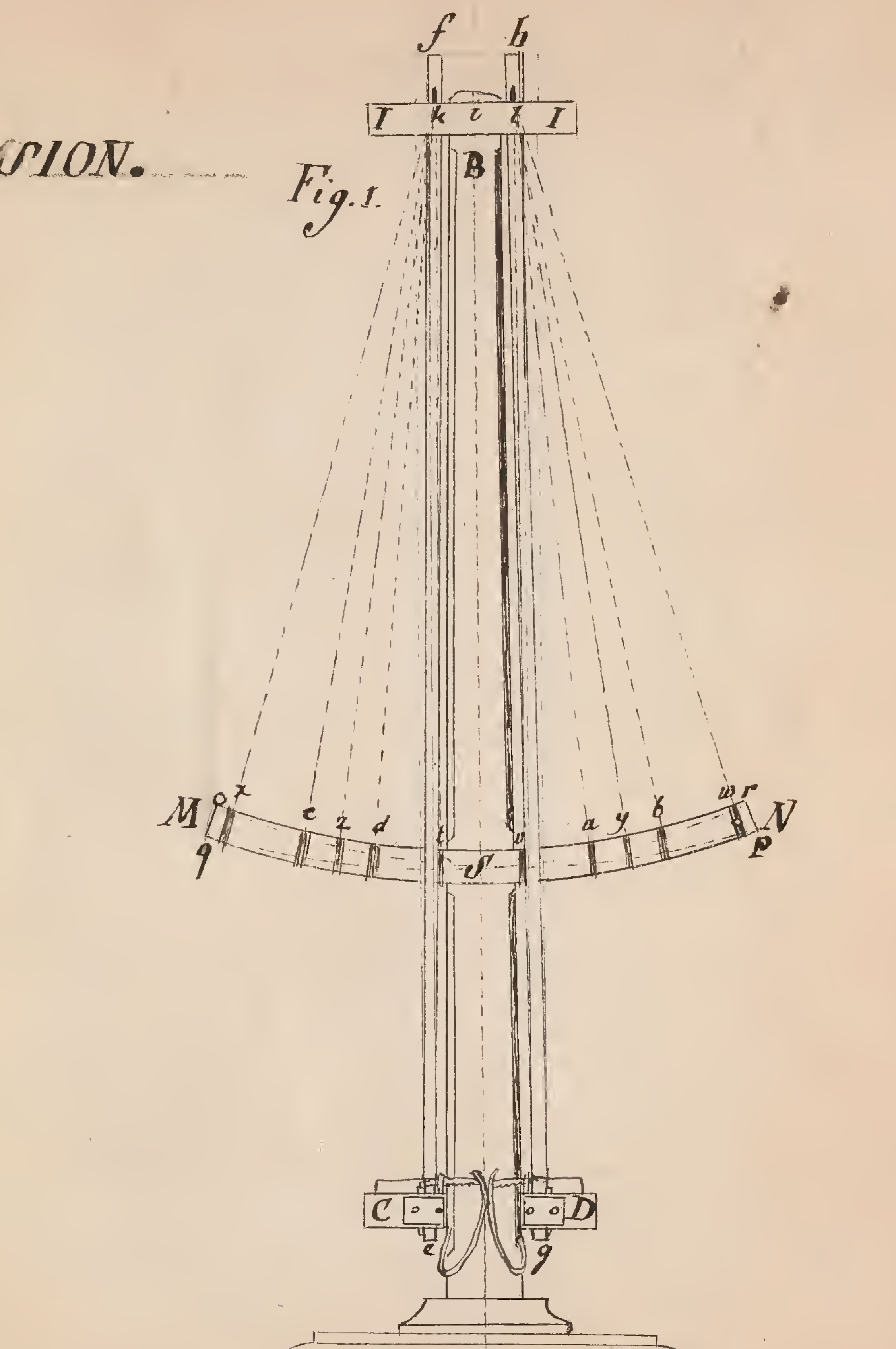
At the outset of the striking body, the center of gravity of the two bodies in our case will be exactly in the middle between the two; and where they meet it will have moved from their half distance to their point of contact, so the velocity of the Center of Gravity before the bodies meet will be exactly one half of the velocity of the striking body; and, therefore, if the velocity of the striking body is 2, the velocity of the Center of gravity of both will be one. After the stroke, as both bodies are supposed to move in contact, the velocity of the center of gravity will be the same as that of the bodies; and as their velocity is proved to be the square root of 2, the velocity of their center of Gravity will be increased from 1. to the square root of 2: that is from 1. to 1.414, &c.

The fair inference from these contradictory conclusions therefore is, that an unelastic hard body (perfectly &c) is a repugnant Idea, and contains in itself a contradiction; for to make it agree with the fair conclusions that may be drawn on each side, from clear premises, we shall be obliged to define its properties thus; that in the Stroke of unelastic hard bodies they cannot possibly lose any mechanic power in the stroke; because no other impression is made than the communication of motion; and yet they must lose a quantity of mechanic power in the Stroke; because, if they do not, their common center of gravity as above shewn, will acquire an increase of velocity by their Stroke upon each other. ——

In like manner the Idea of a perpetual motion, perhaps, at first sight, may not appear to involve a contradiction of terms; but we shall be obliged to confess that it does, when, on examining its requisites for execution, we find we shall want bodies having the following properties; that when they are made to ascend against gravitation their absolute weight shall be less; and that when they descend by gravitation (through an equal space) their absolute weight shall be greater; which, according to all we know of nature, is a repugnant or contradictory Idea. ——

OLLISION.

Fig. 1.



4
1 Part

Machine for COLLISION.

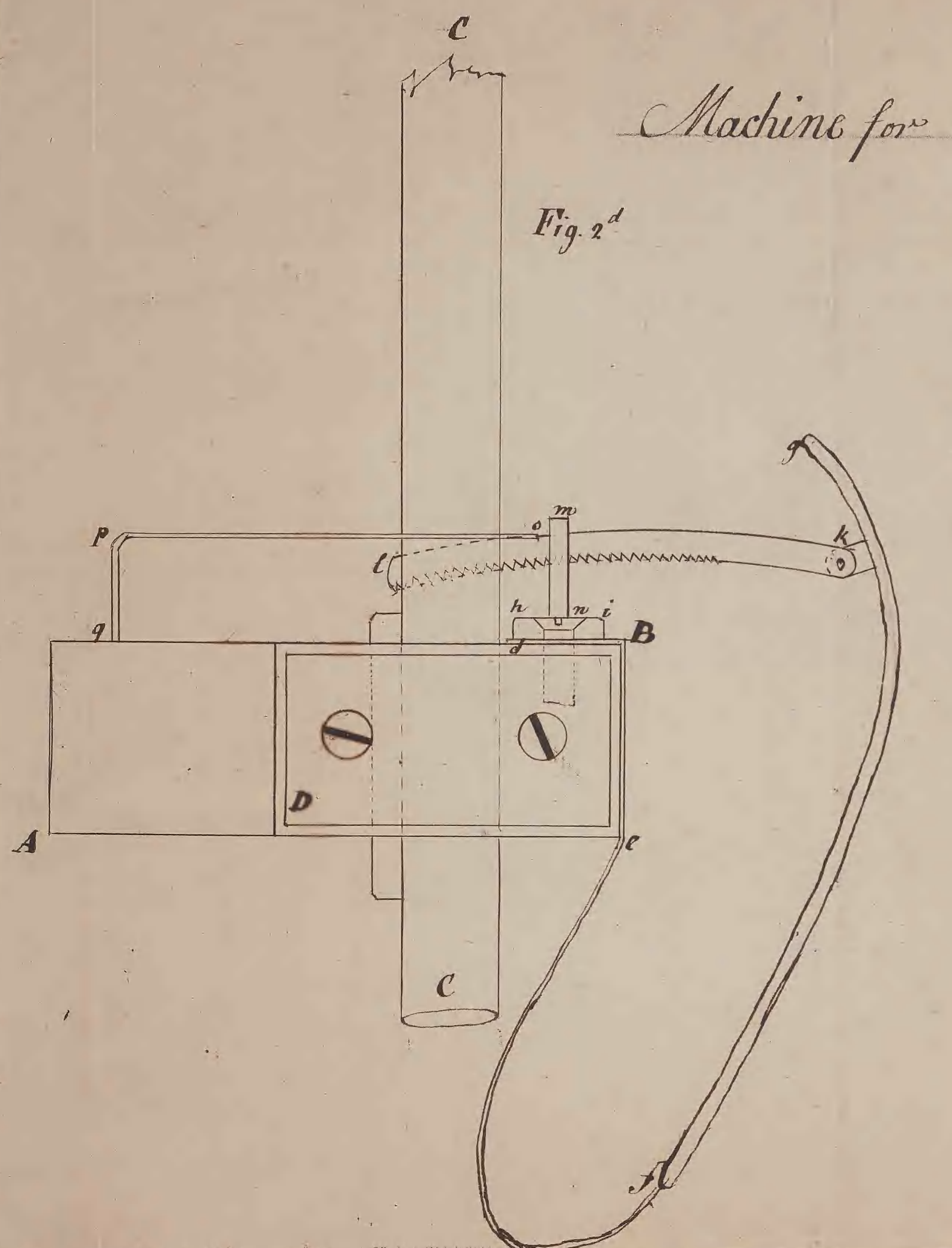


Fig. 2d

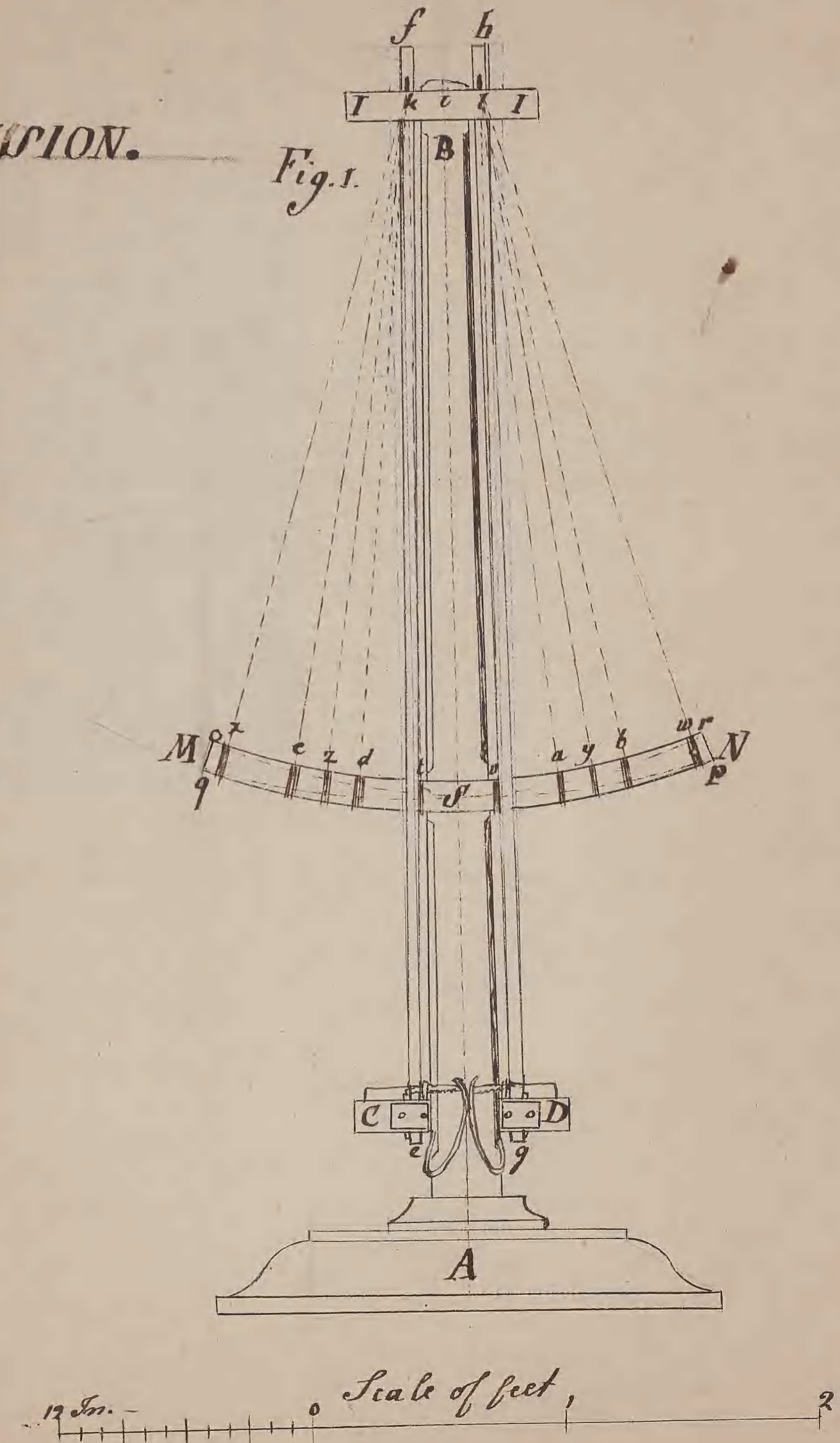


Fig. 1.

It is one of the principles adopted in the Laws of Motion that "when a body in Motion strikes a body at rest both bodies "proceed forward with a velocity which is to the Motion of the "moving body ~~inversely~~ as the quantity of Matter in both Bodies "is to the quantity of Matter in the moving body" or in other words " That the Momentum of the two bodies is exactly equal "to the Momentum of the Moving body before it struck the "other body at rest" From which it follows that there is no force lost in the communication of Motion from one body to another

This is undoubtedly the case where the two bodies are perfectly hard & altogether nonelastick but what follows is intended to show that it is by no means the case with bodies that are not so - and as there are no bodies in this Earth that possess these properties all Machines & calculations founded on that adopted Law of Motion must turn out very different from a True Theory would make them & under different circumstances will vary much -

When the Momentum of the Moving Body is composed chiefly of velocity and acts upon a body incomparatively heavier the above mentioned Law does by no means hold - that is when the quantity of Matter in the moving Body is small and its velocity great and the quantity of Matter in the Body to be put in Motion great - To examplify this in the first place let it be supposed where the last

mentioned properties take place in a very high degree that is Let the Body in motion be very small its velocity very great & that of the body to be moved comparatively great —
Thus suppose A is a Leaden Bullet weighing 1 lb moving with a velocity of 1000 feet ⅌ second against a suspended body B which weighs 99 lbs Then should the Bodies A & B move forward together with a velocity of 10 feet ⅌ second, But experience has always shown that it will not be so for were the Body B a soft supposed wood the Body A which moves with the velocity of a Common Ball would pass through without com-municating any sensible motion to the Body B, though it is well known that there is no method by which the hole through which A passes could be made without exerting a force on the body B sufficient to move it with a very con-siderable velocity and if the bodies were both hard it is equally well known that if the bodies were both hard the body A will be recoiled with a considerable velocity while the body B will remain to all appearance unmoved - From this it would appear that a certain time is necessary for the communication of motion and that the larger body requires the longest time - To make this more clear let the following fact be considered -

When a Ball is discharged from a Cannon the inflamed Powder acts like a Spring in seperating the Cannon & Ball and the Ball is discharged with a velocity of 1200 feet ⅌ second while the Cannon recoils with a velocity of only feet ⅌ second which is not above of the velocity with which it ought

to Recoil and let the Ball Strike on a Body equal in weight & Similar to the Cannon & its carriadge from which it was discharged then ought that body to be moved forward with very nearly the same velocity as the Cannon recoiled but it will not be so for the Body Struck in that manner will scarcely be moved at all altho' there is a force exerted on it nearly equal to the force that caused the recoil in the cannon & its carriadge

From whence it follows that the larger body received the lesser velocity and that the same body received a lesser velocity when acted upon for a lesser time for the Cannon and its Carriadge that were Struck was acted upon by the Same impelling force but in a much Shorter time than the Cannon from which the Ball was discharged for the Cannon from which the Ball was discharged began to be acted upon as soon as the Powder began to be inflamed and continued so till the Ball was quite out at the Muzzle which though a time not sufficiently long to admit of its getting a momen tum equal to that of the Ball yet was much longer than the time during which the Ball acted on the other Cannon and therefore the Recoil was much greater

Now to give another example when a staff of wood strikes with great velocity on a Stone the stone is moved forward with a considerable velocity Let the Same Staff Strike a Ball of Lead and it will be moved forward with a less velocity but suppose the Staff to Strike an Elastick Ball it will send it off with a prodigious velocity and with a momentum much greater than that of the Stone or Lead the probable reason for which is that

the Lead neither being hard nor Elastick the blow that it receives makes an impression on the Ball instead of moving it at Least part of the force is employed in Making that impression The Stone being somewhat harder & More Elastick does not admit of the force being expended in Making and impression & therefore moves with a greater velocity The Elastick body receives an impression which its elasticity gives it an opportunity of returning and so renders the time on which it is acted greater and therefore ~~and~~ the time that the Staff acts upon it although small in itself is yet Much greater than it was when acting on the Stone or Lead. —

When a Body at best not perfectly hard is struck by another Body in Motion the first thing that takes place would appear from what has been said above is a Motion of all the particles or Minute parts on one another and this is greater or less according to the nature of the Body the velocity with which it is struck Whatever force is exerted in moving the particles on one another is lost for it cannot also be employed in Moving the Body unless it is perfectly Elastick and regains its shape with a force equal to the force required to Make the impression and therefore unless a body is either perfectly hard to admit of no Motion of its parts or perfectly Elastick to lose nothing by that Motion it cannot receive a momentum from any other Body equal to the

Momentum lost by the other Body in giving it and this loss will be greater or less in all Bodies according to their nature & the proportion of the velocity to the quantity of matter

From this have undoubtedly many of the mistakes in the Calculations of Mills taken place and perhaps the great dispute that has subsisted many years among Philosophers - "Whether the Momentum or Forces of Bodies "are as their quantities of Matter & velocity Simply or "as their quantities of Matter & Squares of their velocities" For it is very evident that were two experiments to be made one with a small body & great velocity acting on a Larger one and the other with a large body & small velocity acting on a Smaller one or on an equal body the conclusion that would follow in the two cases would be exceedingly different even if the Matter in the bodies were the same in both cases but much more would it differ if it happened to be tried with Elastick bodies in the one case and nonelastick in the other — And it would also turn out very different if the Momentum of the moving Body were estimated by penetrating or by giving Motion to the Body at Rest - For two Bodies with equal quantities of Momentum will produce very different effects - one body of 1000 lbs weight with a velocity of 1 foot pr Second will remove another of 1000 lib weight while another of one pound weight with ~~one~~ 1000 feet of velo- will give it no perceptible Motion Now in this

case if the effect were Estimated by the motion given to the Body at rest. The large body would be found to have hady by much the greated momentum but if on the contrary it were computed by the alteration made in the form of the other Body the Small body would ~~have~~ be found to have had the greated momentum now as their do nat in fact exist in this World any Bodies hard & Elastick no Experiments can be convincing till the Law by which these Soft & imperfectly Elastick bodies are governed be first determined. —

A Memorial.

Upon the centering and decentering of Bridges and upon the diffirent figures that the arches assume during their construction by Monsieur Perronet. 22nd April 1773

The construction of great Arches such as those of many Bridges made in France within these thirty years, demand much more art and care than those at or under the middle size. Independant of the choice of materials, the exactness of their preparations and the care with which the stones are wrought and laid, the success of these large Arches depends essentially on the manner of centering and decentering them, without such attention it often happens that the Arches have lost their figures and even some of them fallen. These considerations which are so interesting to works of so great importance have appeared to me to merit the attention of the Accademy, the public, and the Artists which have the making of their project and constriuction.

I have proposed to myself in this memorial to shew

I. what appears to me the most convenient way of making Wooden Centers for the construction of Stone Bridges.

II. The different Motions that the Arches assume during their construction, a very interesting subject and has as yet never been treated of.

III. The method which I have employed with success for the decentering the greatest Arches.

Centering of Bridges.

For the construction of Stone Bridges in general we are obliged to make use of a piece of wood work called a Center, or by the Italions an Armature, which must be sufficiently strong to support the Arch untill the Key Stone is drove; this wooden frame is composed of pieces of wood placed vertically, called fermes or Ribs which are commonly placed at the distance of six or seven feet from one another, and horizontal pieces called Stays or Sleepers, which are destined to carry in their middle. each course of Voussoirs between the Ribs we put strong frames or pieces of wood under these sleepers and little pieces above them to keep up or support each range of Voussoirs, according to the hight that the curvature of the Arch requires; the Ribs are afterwards bound together by the moises (1) & the liernes (2) placed horizontally and kept together with pieces placed across from the one side to the other to prevent them from seperating.

The fermes are commonly made with horizontal pieces called **entraits** of Arbaletreirs of poençons of moises pendant and hanging posts the whole joined together with Mortises, tenants & pined with Iron pins. you will see in memoirs of the Accademy of 1767 the design of a ferme which has been given

[I.] There are pieces which embrace jointly the other pieces of wood

[II.] There are other pieces which are checked or notched a few inches into the other pieces to keep them from shifting.

By Monsieur Pitot for semicircular Arches and under 60 feet opening; the entraits and even the Arbaletreirs will,

Accor-

will according to this design be charged or loaded laterally, which is the most disadvantageous manner we can apply wood and of consequence we must increase the quantity to carry the same weight.

When the centers are only supported against the piers and piles of the Bridge we call them retrograde firmes. each point of support can then be placed upon one piece of wood alone a corbel or pier in place of being upon many rows of stakes as we have been often used to do

The tenants & mortises weakens the wood, we ought to prevent it as much as possible, in putting principle pieces of the frames called Arbaletriers, upon many rows or ranges in joining the one above the other in such a manner that the ends of one of the ranges be below the middle of the superior Arbaletriers with which they form triangular figures, which will have for their base the entire length of one Arbaletrier and for their sides two demi Arbaletriers of the range above, the principle pieces ought to have Moesies in the middle of their length just as the others are at their extremity and pinced.

This manner of disposing the wood of frames or centers, which has been done by M. Mansart de Sagonne at the Bridge de Moulins, appears to me the most propper and I have adopted it in leaving out nevertheless many pieces of wood that appeared to be unnecessary.

The centers will sink after being put together by their own weight, as also by that of the arches during their con-

struction

struction on account of the fibres of the wood compressing whereby the Arbaletriers will be a little curved and of consequence we must keep the center as much above the intended curve of the Arch as will make it after having sunk the intended curve of the arch, which sinking must be learned from experience

I will now explain the principle dimensions and the joinings of the fermes which I have caused to be constructed for Arches of 60, 90 & 120 feet of opening, afterwards the observations which I have made on that subject.

Arch of 60 feet opening

The middle Arch of the Bridge de Cravant, situated upon the River d'Yonne of 60 feet opening, and 20 feet high under the Key Stone from the spring of the arch, has been centered with five retrograde fermes, placed at the distance of 5½ feet from middle to middle, each ferme composed of three courses of Arbaletriers, the first and the third forms of five pieces in length and that in the middle of four. These courses of Arbaletriers were placed so that the middle of the one should be above the joinings of the other, forming triangular spaces and kept by the Moises as has been before explained; each Arbaletrier was from 15 to 18 feet long and from 8 to 9 Inches thick; the Moises were the same thickness and from 7 to 7½ feet long; the thickness of each course of Couche was from 4 to 5 inches. The Stone employed at this bridge, weighed 176 Livres the cubical foot and the thickness of the arch was 4 feet at the Key Stone.

Arch

Arch of 90 feet.

The Arch de Saint-Edme constructed at Nogent-Sur-Seine and finished in the year 1769, was 90 feet wide and 26 feet high from the spring to the Key Stone, it was Centered with five retrograde fermes, placed at the distance of 7 feet from middle to middle each ferme formed of three courses of Arbaletriers, as in the preceeding Bridge; the first and the third firm was made of 5 pieces and that of the middle of 4. these pieces were of 18, 20 and 22 feet of length and from 14 to 16 inches thick; the Moises were the same thickness as the Arbaletriers, and from 7 to 8 feet long, each course of Couche was from 7 to 8 inches thick.

These Centers were of great strength; I believe it would have been sufficient to have made the Arbaletriers from 12 to 15 Inches thick as they were in the device, whereas the undertaker has given them from 14 to 16 inches of thickness in order to answer the wood such as he had been able to find in the forests.

The Stone of which this Bridge was built weighed 180 livres ℔ Cubical foot, and the thickness of the arch at the Key Stone was 4 feet 6 inches.—

Arch of 120 feet.

Each of the five Arches of the new Stone Bridge at Neuilly, was 120 feet wide, and 30 feet high under the Key Stone to the Spring of the Arch, and 45 feet across from side to side, it was centered with 8 retrograde fermes, placed at the dis-

tance-

tance of about six feet from middle to middle; each ferme was composed of four courses of Arbaletraiers binding or strengthening each other by being disposed in a triangular manner, as were those of the two preceeding arches. The two outside fermes were composed of eight pieces; the second and fourth to these each of seven pieces, and the third of six pieces, the whole of which were from 19 to 23 feet long, and from 14 to 17 inches thick; the Moises pendants, to the number of 13 was from 9 to 10 feet long, and from 9 to 15 inches thick each; the whole was bound together with 5 horizontal Moises from 9 to 15 inches thick, and 8 liernes of 9 Inches of thickness; the Couches or sleepers were from 7 to 8 inches thick; the calles of the upper and undermost of the Couches was from 6 to 7 inches the one, and the other, which was that of the sleeping or lying one, was about 2 inches thick: in this manner the intervale between the upper side of the fermes and the arch was from 17 to 18 inches, being necessary to give it double the hight of the Sleepers this distance is even augmented or encreased during the setting by 6 or 8 inches, by the sinking of the fermes which we are obliged successively to heighten to these calles. The Centers of the middle arch of the new Bridge of Mantes, which was also 120 feet wide, were also retrograde, and I have given to the pieces of wood the same disposition between themselves and nearly the same thickness as the fermes of the Bridge of Neuilly

The Stone which was used at this Bridge and that of Mantes is mostly from the Quarry of Saillancourt near Meulan; the weighed 165 livres ℔ Cubical foot, a little more or less according to the different Strata's; the thickness of the arch at the Key

Stone

Stone was 5 feet.

For the better understanding of what follows upon the centering of these different Arches we have subjoined to the present memoire the design of the forme of each of each ~~of these arches~~

Different motions the arches took during their construction

We begun with laying the first courses of Arch stones without the Centers even untill they begun to slide upon the under ones; this ought to happen nearly as soon as a Stone will that is placed upon a Piece of sawed wood that is not planed as I have observed in my memorial inserted in the Volume of this accademy of the year 1769; when the upper part of the Stone is inclined with the horizontal about 39° or 40° in place of 18°-20° the angle of friction of little polished bodies: I say little bodies, because this angle is reduced to about 4°; for large bodies such as Ships which are launched into the Sea, upon a plane to which we give this small inclination as I have said in the same memorial.

The course of Stones or Voussoirs that we lay afterwards on each side, begins to press upon the Centers; this weight or pressure is augmented successively untill the Key Stone is drove, and by pressing down the parts where they are laid tends to make the top rise; on this account we generally take the stones which are intended for the top of the arch and lay them on the Center to prevent

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it from raising.

The weight of the Stones that were laid upon the Centers of 60 feet was 67500 livres there being at that time 13 Courses of Voussoirs laid, making a seventh part of the whole for each side, the Centers were not raised; the sunk only one inch under the weight of the arch.

The whole weight of the arch before the Key Stone was laid was about 1000350 livers, and that weight ought (according to the calculation of M Couplet and related in the Memoirs of the Academy, anno 1729) to be reduced by the $\frac{4}{9}$ of the whole or about 600000 livres, for the weight which with the Centers are loaded, and to 120000 livres for that with which each firme is loaded.

The weight of the top of the Centers of the arch of 90 feet was about 350000 livres; we had laid then the 15th Course of Voussoirs making nearly the sixth part of the whole arch on each side; the Centers which had been raised only three inches more than the curvature of the arch, sunk at first two inches under the above weight and afterwards raised one inch; when we had laid the twentith part course of Voussours, it flattened a little in the reins; when the arch was about three fourths built the Centers still lowered one inch & a half by the alone compression of the wood, without regard to what we have remarked respecting the swelling or straightening of the reins, and of three lines only more under the whole weight: then there remains no more than 3 lines of rising of the 3 inch that whe had given to these Centers.

Thus

Thus the whole weight on the Centers before the Key Stone was drove, being reduced, as I have explained before, ought to amount to 1040000 livres and that of each ferme to 249000 Livres.

As for the arches of 120 feet of the Bridge of Neuilly, we began at the end of the year 1771 to load the top of the fermes with 52 Voussoirs of the weight each of 5000 livres the whole weighing 260000 livres; the were compressed by this weight only 9 lignes, and was not more during the whole winter; there was then 18 and 19 Courses of Voussoirs laid on each side of the arches.

The 7th of July 1772, the weight on the top of the Centres and the greatest weight that was laid, was 186 Voussoirs which weighed about 900 Milliers or Thousand weight, independant of these, there was 46 Courses of Voussoirs laid on each side, the sinking has not been more than 19 lignes.

About the 26th of the same month we had finished the laying of the Key Stone, and then the whole sinking which was increased sensibly each day under the weight of the last twenty courses of Voussors, was found to be 13 Inches 3 lignes.

The whole weight on the Centres was for each Arch before the Key Stone was laid 2 Millions 400 thousand livres, and for each of the eight fermes, 300 thousand livres.

This inevitable sinking of the fermes occasioned at first an opening in the upper sides of the joints of the Voussoirs at

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a little distance from the perpendicular above the spring of the Arch (particularly in great arches) and afterwards successively higher, to that proportion we raise the Arch; which frightens persons that are not acquainted with these kinds of constructions, that these effects are not occasioned by want of care, and cannot hurt the solidity; but that the joints shut themselves after that the Key Stone is laid: which I shall explain in the last part of this memorial in speaking of the decentering of arches.

As for the Arch of 60 feet of which I have spoke we perceived not that motion during the laying of the 18 Courses of Arch Stones on each side; the effect was so inconsiderable The Arch of 90 feet being raised from the 20th on each side to the Eighty fifth course of Voussoirs which composed the arch, the joints opened on the upper side even to 9 lignes from the 15th course of Voussoirs, crossing the solid parts of the Vault at the reins near to the perpendicular above the spring of the arch which occasioned a seperation vertically on the backside of the Voussoirs in descending even to the seventh course, with the running and horizontal course in the arch next the land.

A little time after, these joints having begun to shut themselves, the opened others a l'extrados to the height of the twenty sixth course and even to the thirty first course of voussoirs each nearly a ligne from the one part to the other of the Arch.

As to the joining of the land Arch of the Bridge of

Neuilly

Neuilly the joints opened themselves to their extrados from the twelfth even to the thirty sixth course of Voussoirs on each side, and 1/4 of a ligne even to two and three lignes, except those between the 26th. and 27th courses of Voussoirs which opened 10 lignes at the arch next the land on the side of Neuilly, and that of the other land arch only 6 lignes; these openings were in the land arches: the openings were less in the others.

A little time after the laying of the Key Stone, the joints l'intrados or lower side of Voussoirs, opened themselves upwards from the thirty sixth course even to the fifty sixth, which joined the Key Stone, from 1/4 of a ligne to a ligne, but only in one two or three joints or more in each arch.

As for the Bridge of Mantes, whose middle arch was, as I have said before, of the same width, viz. 120 feet and 35 of height under the Key Stone, the joints opened nearly as those of Neuilly.

Decentring of Bridges.

To diminish the sinking of the Vaults, and to facilitate the decentring of Bridges, the common custom has been even to the present, to lay a certain number of the last courses of Voussoirs dry and to bind them strongly together with wooden wedges drove with a mallet between them in the joints soaped and to run & futch them afterwards with lime morter and Cement: we did not however do it at the Bridge of Neuilly because I thought that the percussion

of

of the strokes of a Mallet would have little effect to squeeze or firm the Voussoirs between themselves as they were such great masses of stone each being at least five thousand weight and some of them even eight or ten thousand; I besides was afraid of breaking the Voussoirs, as it has happened at other Bridges in driving these wedges which are often not bearing perpendicular on account of the difficulty that we have of placing them over against each other.

Some Engineers are in use of leaving the Arches as long time as they can upon the Centres; others strike them immediately after they have made them firm.

When we have time at the end of the year, we leave them standing a month or six weeks; but it is always prudent to leave the Centres untill the Morter is so fastened in the joints as to admit with difficulty the edge of a Knife, which will happen in 15 days or 3 weeks and particularly if the stone is dry and porous for then it will imbibe more quickly the moisture of the Morter.

The decentring of the Bridge de Cravant was begun fifty days after the Arch was finished and made in a few days le tassement of the Arch was insensible.

The fear of being surprised in the latter end of the season by the Spates, obliged me to begin the decentring of the Arch de Nogent Sur-Seine; three days after its fastening; this interval of time way employed in driving wedges into the 13 last courses of Voussoirs and in running and pitching them. The Morter however would have required a longer time in that Bridge to acquire a certain consistance on account of the hardness of the

the stone that was made use of; but depending on the security of the method that I had adopted I thought I would run no no risk, and that there would result nothing but a greater tassement of the Vault, which tassement would even become usefull to diminish the slope or declivity of the Bridge.

The decentring was effected in five days, in the manner which I will explain afterwards in speaking of the Bridge of Neuilly and which was also employed at that of Cravant.

The fermes which was compressed only 2 Inches 9 lignes under the weight of the arch, rose again two inches after the taking away of the sleepers and consequently the straitening from the underside of the Voussoirs and the elasticity of the wood rebounding again.

The second day after decentring, the joints which had opened on the underside of the arch as I have said before contracted two lignes; the third day the greatest joint which was situated on the side of the City opened of itself three lignes; two hours after the taking away of all the sleepers, these great joints were entirely firm on the side of the City and was two lignes nearly on the opposite side; those on the superior part of the Vault also contracted of themselves.

Le tassement total of the Vault was (45 days after the beginning of the decentring) 12 inches 6 lignes at the Key Stone, distributing itself proportionally upon the other Voussoirs even to the seventeenth course; below these courses of Voussoirs, the curve raised in the place where it had sunk upon the centers during the construction of the Vault.

Vault, this change upon the whole was so regular that the curve presently became very agreeable to the Eye and without any inequalities; it happened that the superior part of the arch was at a radius of 123 feet instead of 100 which it was before according to the original before the flattening of the arch: Le lassement was augmented 15 lignes the first year in such a manner that it was actually 13 inches 9 lignes at the Key stone.

To render this change of the curvature more sensible, and in order to distinguish the parts of the Vault which tended to overturn the abuttments and the piles of those of the inferior parts which resisted this effort, I made draw or cut before the decentring a horizontal line upon the Voussoirs from the head of the arch to the 28th course counting upwards, and another oblique line directly to the reins from the extremities of this horizontal line to the place where they join with the seventh course of the wall in the widening of the abuttments.

The horizontal line has made known by its curvature, that of the lowering of the corresponding Voussoirs in adding thereto that of the extremities which we had reffered to a fixed point

The oblique lines were curved with inflexion in such a manner that above the 17th course of Voussoirs it was convex downwards, and below that Voussoir it was concave downwards; the greatest ordinate was 6 In: 10 lignes in the middle of the convex part and 5 inches 6 lignes at 2/3 from the concave part reckoning from below—

This point of inflection which ought to seperate the

two

two actions which act in contrary directions was besides rendered more sensible by the joint which was opened in this place.

The little arch which terminated above from the 17th Voussor was 50°, it comprehended almost exactly 1/3 of ~~the~~ half the Vault.

The Knowledge of this point of inflexion is very important for the theorie and the calculation of the Shoot of Vaults and with like observations made upon Arches of different magnitudes and curvitures we will be enabled to establish formulas with more certainty as M de la Hire (1), Couplet (2) and d'Anesy after the Hypotheses with which the have been obliged to content themselves, made from like observations.

There remains only now to give an account of the decentering of the Bridge of Neuilly, which required the greatest precautions on account of the hardiness of the undertaking.

I have said before that the fermes are garnies de couchis avec leurs calles which carried the courses of Voussoirs: it was these calles & couchis which it was necessary to remove slowly and in a certain order to detach the fermes from the Arch and to leave them standing by themselves. afterwards there will be nothing to do but to take down the fermes and to finish the decentring. I have said also that we ought to consider two parts in an arch, a superior one which tends to descend an inferior one on each side which resists and tends to push the other upwards: this last

(1) Mem. de l Acad. annie 1712.
(2) Idem annie 1729

part ought to comprehend (on each side of the Vault) that which did not bear on the Centres before the Key Stone was laid

M. Couplet having made the search for the part of the Arch where the Voussoirs began to load the Centre before the Key Stone was laid, has found in supposing that the Voussoirs were polished and without friction that there would be in a semicircular Vault 30 dgrees or 1/3 of half the circumference that would not load the Centre.

We have seen in the Arch de Nogent-Sur Seine that the part of the Arch which has been pushed outwards and which of consequence out not to load the Centres before the Key Stone was laid, was equal to 1/3 of half the circumference.

As to the Bridge of Neuilly the curvature of the head being of one Arch alone supported by a sett of Voussoirs bended like a Cows horn, the inflection of which we have spoke before was not remarked but the greatest joints made their appearance above the 26th course of Voussoirs which ought to make on both sides the seperation between the superior parts of the vault which tended to repel the inferior parts, and this point is two Voussoirs above the middle of half the Arch that which in these Arches agrees nearly with M. de la Hires Hypothesis.

We can then after these observations begin to remove without much uneasiness all the sleepers which are laid on both sides downwards in the vaults, all at least from 1/2 of half the Vault since when the Key Stone was laid, these parts in place of bearing on the Centres was

pushed

pushed outwards by the weight of the superior voussoirs; we were the more confirmed in this opinion when we were lifting the Calles & les Couchis which were in these parts holding or bearing so little weight; that we found many of them detached below the stones when we were going to raise them.

We ought in the mean time to be carefull in raising these sleepers slowly and taking many days and removing them in equal numbers every day and on each side at the same time in order that the fermes be able to repel the superior weight occasioned by the vacancies left by these sleepers and consequently prevent the vault from descending quickly because we ought with the greatest care to prevent so great a mass from acquiring ~~so great~~ a certain velocity, it is only by moderating this velocity even untill all the sleepers are removed from below the vault that we prevent any fractures in the stones, and even the endangering of the arch itself if we used it otherwise.

These observations out to make us abandon (principally for vault made with turned uper retrograde Centres) the ancient custom which was to remove the sleepers in pairs equally from each side throughout the whole vault, and to continue the operation untill all the sleepers were removed; now we leave by this method points of support under the superior parts of the arch which prevent the uniform and general sinking and occasions often angles of irregularities in the curvature

of

of the Vaults above all in great arches, which are even exposed to the greatest accidents when they are joined to any defect in the construction.

Things being as we have explained, we begun 14 aug. 1772, 18 days after the laying of the last Key Stone of the Bridge of Neuilly to remove the sleepers du bas des Vault to comence from the 9th course of Voussoirs, those from below having been laid without sleepers; we have continued afterwards even to the 3.d of September following to raise the rest of the Sleepers in equal numbers every day on each side and afterwards ascending and leaving some days of interval at different times without working there, in such a manner that all was raised in 19 days: which was done, observing however to put des'etrésillons or little pieces of wood set in end, between the fermes and the Vault, to facilitate the removing of the calles and superior Sleepers as we observed the fermes beginning to rebound by the elasticity of the wood and consequently prevent their removing. There remained only the last day seven courses at the top of the fermes, which I made be removed: les'etrésillons having been ruined that is distroyed by a Chisel and Mallet, the whole was distroyed in less than an hour, which was done at the same time to all the arches.

The Carpinters began with the range des'etrésillons the most removed from the Key Stone, and drawing nearer it in running always at the same time on each side the superior ranges, when they had arrived at last range we saw

these

these étrésillons crush of themselves with considerable force and he that directed this operation at one of the arches was overthrown with the burst or crushing of one of these étrésillons which came and struck upon the reins. The fermes which was found till then 19 inches lower including 6 inches after the laying of the Key Stone, the whole in place of 15 inches of which they had been over rated raised themselves 5 inches 6 lignes and almost equally at each arch with some force and noise.

The sinking or weighing down of the fermes was not more than 19 lignes the 18th of July after the laying of the 46 courses of Voussoirs on each side and 7 inches 4 lignes under the total charge of 930 thousand livres after the laying of the 55 course of voussoirs.

This lowering was 13 inches after having laid the three last courses of Voussoirs including that of the Key Stone

During the time we were removing the sleepers the vaults lowered 6 inches; the sinking the day we ruined les étrésillons was suddenly 18 lignes and 13 lignes the day following.

Just now the pavement and the parapets are laying on the Bridge, the whole sinking is only 9 Inches 6 lignes and I presume it will not augment above one inch more.

The superior arc of the arches having been measured after the sinking of the vault, we found only 33 feet of a chord from the one point of the corne de vache to the other, la fleche or bending was actually 6 inches 9 lignes of hight, which makes it answer to arch of a circle whose radius is very nearly 244 feet; from whence we see the possibility which we knew not before the construction of the Bridge of Neuilly, of making with

with hard stone and in a convenient place arches of a full semicircle of double that radius or about 500 feet of opening

Before the Key stones of the arches were laid the joints of the voussoirs tended to open of themselves as I have said by the weighing down or sinking of the fermes which motion began at the jambes de force which were placed against the piles and piers to support the fermes and increased in ascending towards the top of these fermes; but the Key stone and the Centres finding themselves soon after unloaded, the cause of the motion of the vault changed and it was from the Clefs and countre-Clefs, from whence departed in contrary directions from the motion of the fermes, the actions of the voussoirs tended towards the piles and the piers which was to support the vaults after the decentring.

It was this last motion of the voussoirs which tended to close the joints again which opened of themselves during the laying of the voussoirs, and those operated more easily when the fermes resisted well by their good joining and strength, the load of the voussoirs.

As to the Bridges at Mantes, the decentring was begun the 10th October 1764, thirteen days after the laying of the Key stone, which employed us 10 days. The whole sinking of the arch was found 15 Months after its construction, 20 inch 7 lignes of which 12 inches was upon the fermes before the Key stone was laid, and 8 inches 7 lignes since that time; which is for the last sinking 2 inches 11 lignes less than the arches which joined to the abutments of the Bridge of Neuilly, in supposing

as

as we have said before that these arches would still sink one inch, this difference might probably arise from the middle arch of the Bridge of Mantes being less elliptical by 5 feet than that of Neuilly

Before coming to the rest of the decentring of the Bridge of Neuilly, I think it will be à propos to observe that these great arches ought to be constructed upon retrograde fermes unless the vaults be of a small height and decentred as as I come to explain, because according to this method the Vaults are supported by the ferms without having their curvature any way broken, and the voussoirs bind insibly between themselves on account of the fermes weakning by degrees they lose as it were their point of support, when we raise the sleepers from the inferior part of the vault against which the fermes were supported, and the vaults continuing to lower insensibly, even untill they support almost themselves, which happens in such a manner that when we come to ruin the trésillons, we perceive sensibly that the fermes carry no more the vaults, and that we could even detach them was it not for the elasticity of the wood endeavouring to rebound.

The King desiring to be present at a part of the decentring of the Bridge of Neuilly, which it was possible to do without any risk of the solidity of the arches, in the little time that his Majesty could give to that spectacle for which he had fixed the 22.nd of September 1772 we had put off the making fall the fermes of the Centres for that

day; after having taken away the Moises the limes and contrefiches which would have annoyed this manouver, and after also having taken down three fermes in each of these Arches situated on the side of the Puleaux, in order that the might not encumber too much by their fall the arm of the River that passed there.

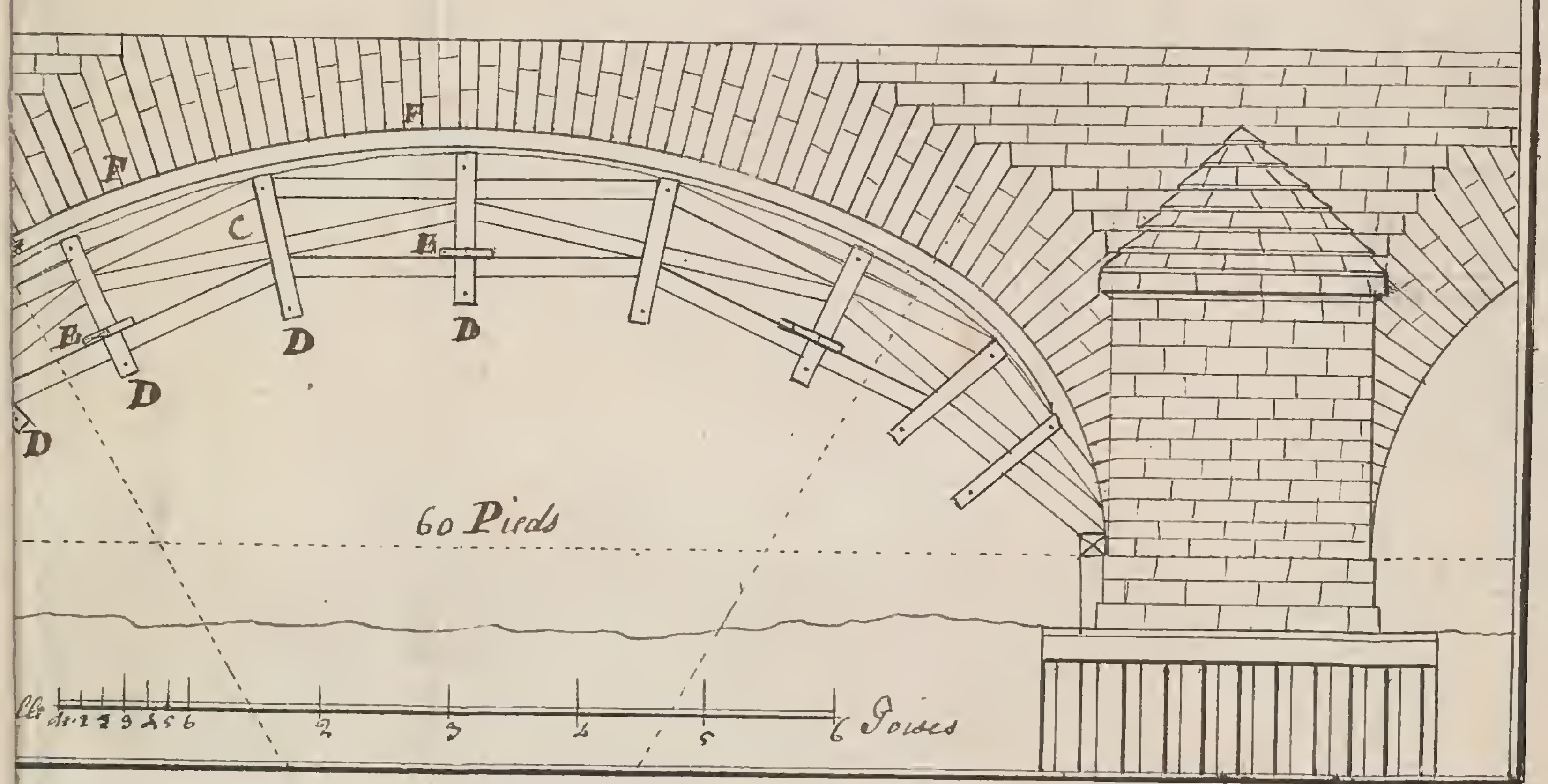
I made be placed two Capestans before each arch and as many behind the two arches situated on the side of Puleaux: these last to make fall ~~the~~ two fermes on the same side, whilst that the remaining fermes of these Arches, and that of the others which were situated on the part of the Isle were to be overturned with the Capestans which were placed for each of these Arches on the side of the ground which was prepared for the King.

The Ropes were fastened to the tops of the fermes and passed over two compound pullies at each end; eight men applyed to the Arms of the Lever, were to to work each Capestan, which was executed at the beat of a Drum and the fermes were overturned in less than three minuthes and a half. The fall of this enormous mass of wood, the weight of which for each of the Arches must be at least 720 thousand weight, made the water rise in foam over to the Bridge. we now saw the vaults at the discovery of which the public appeared affected with a lively and agreable surprise which we beleive ought to be

be attributed to the sudden fall of the which an instant before appeared necessary to support so great an edifice

The precautions we had taken for the construction of this Bridge in the conducting of which I have been very well seconded by M. Chery Engineer of the Bridges, Highways, Causeys &c. and Inspector General of the Streets of Paris, has been attended with the greatest success; we perceived no stones broken nor even a corner defective, nor any joints open, which is as happy as uncommon for so great an Edifice.

ARCHE DU PONT DE CRAVANT.



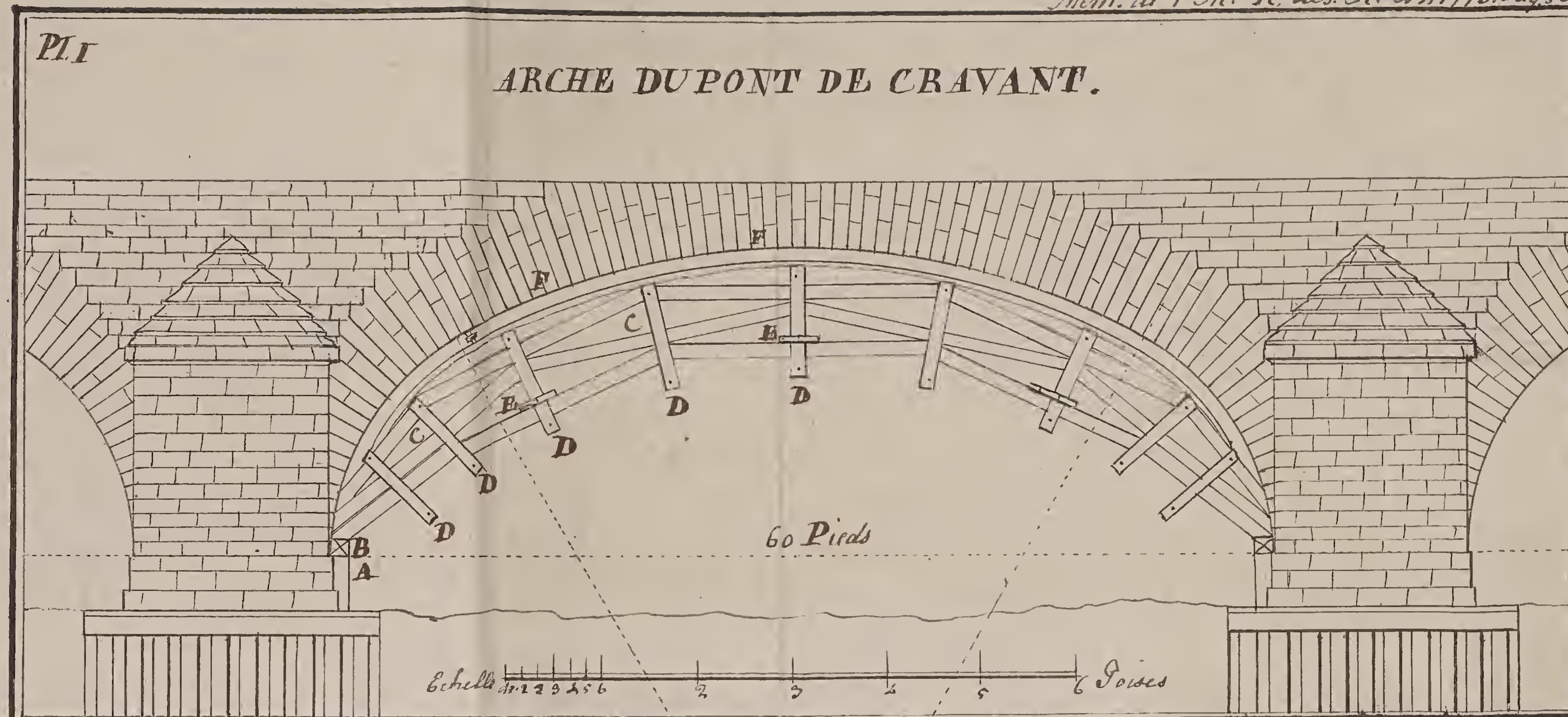
pour les Fermes des trois Ponts.

de Force — D. Moises pendantes

ux — E. Moises Horizontales

striers — F. Couchis avec leurs Cales

G. Liernes. Sur la planche du Pont de Neuilly.



Legende pour les Fermes des trois Ponts.

A. Jambes de Force

B. Chapeaux

C. Arbalestriers

D. Moises pendantes

E. Moises Horizontales

F. Couchis avec leurs Cales

G. Liernes. Sur la planche du Pont de Neuilly.

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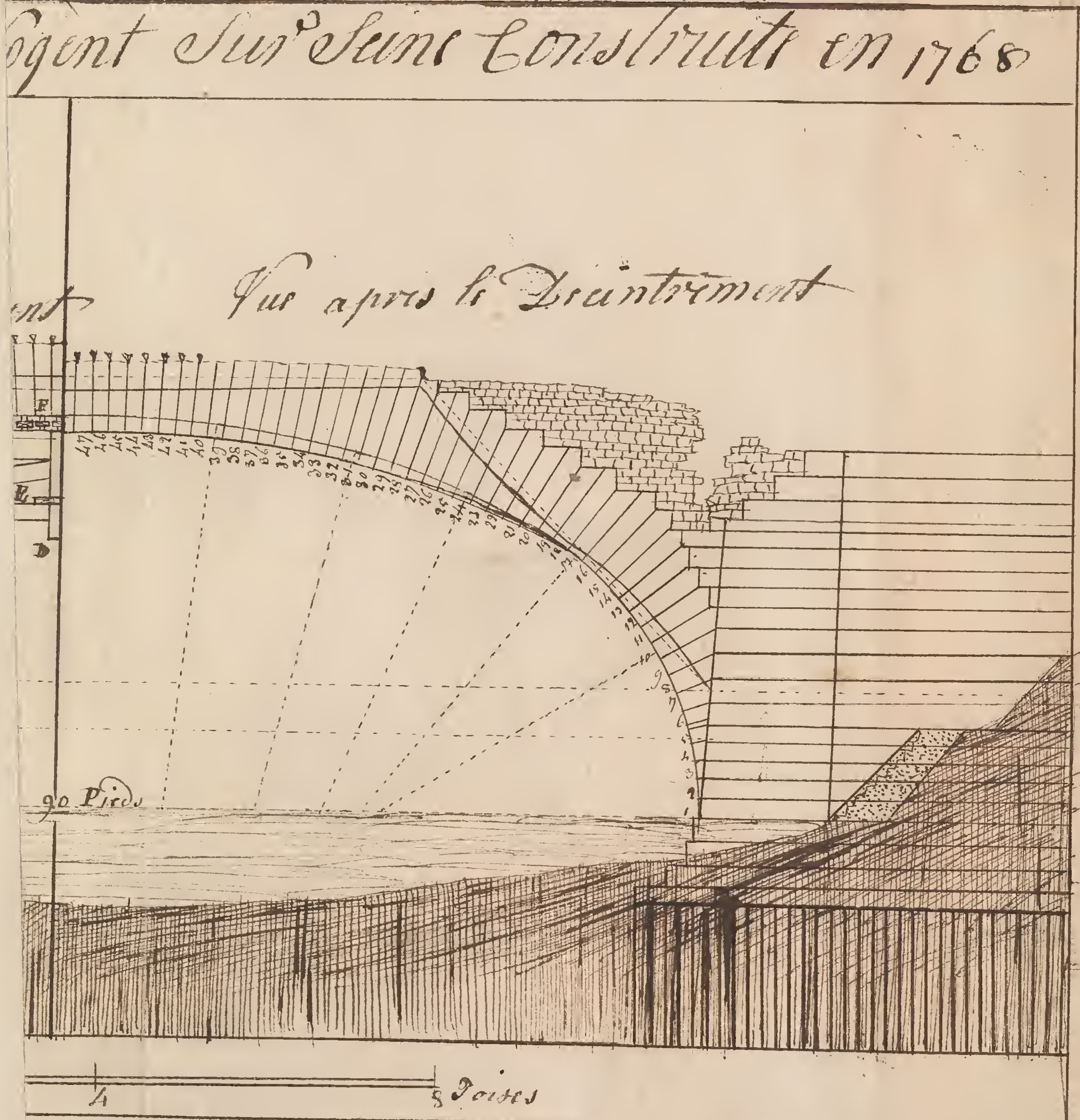
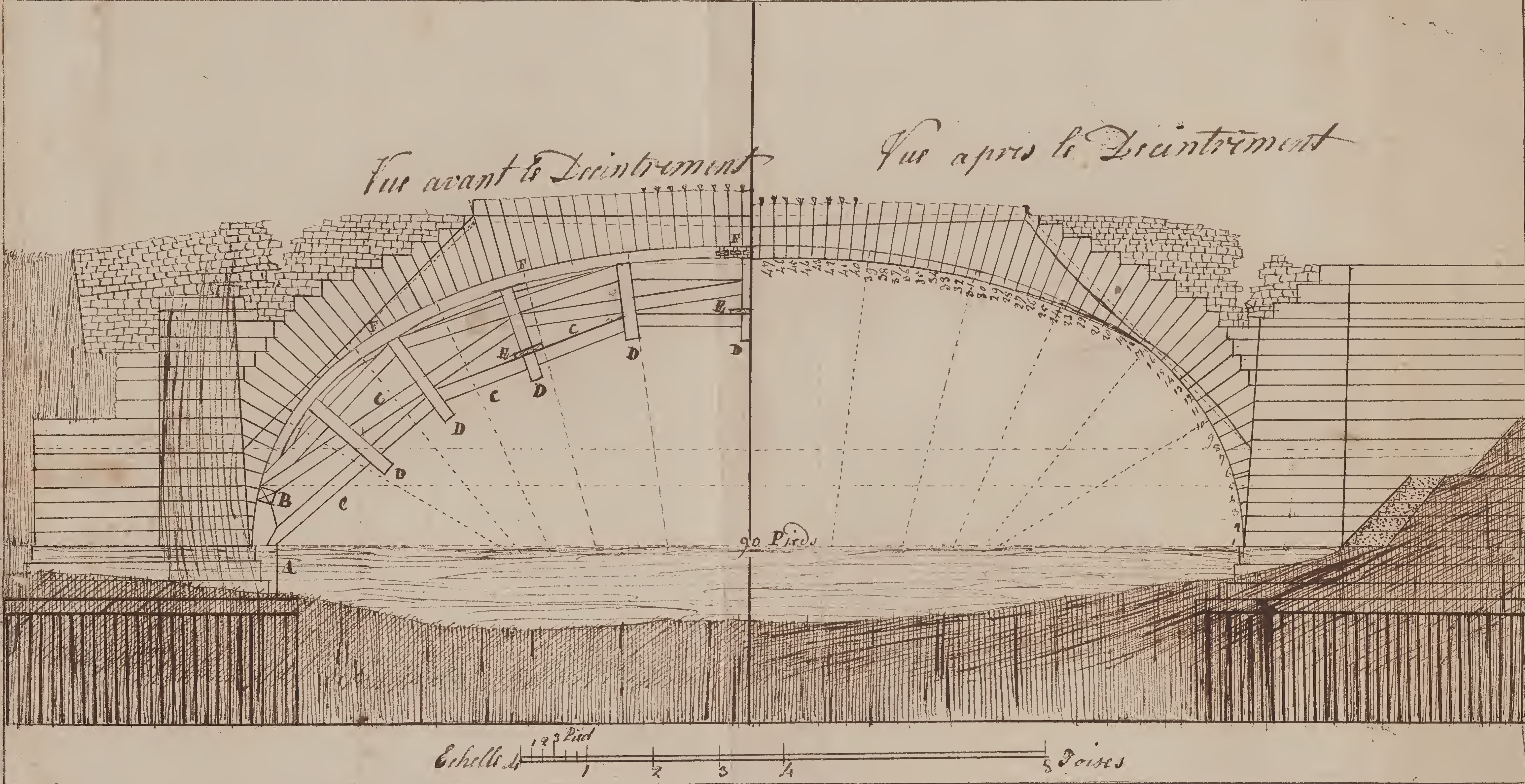


ARCHE St. EDME DE Nogent sur Seine Construite en 1768



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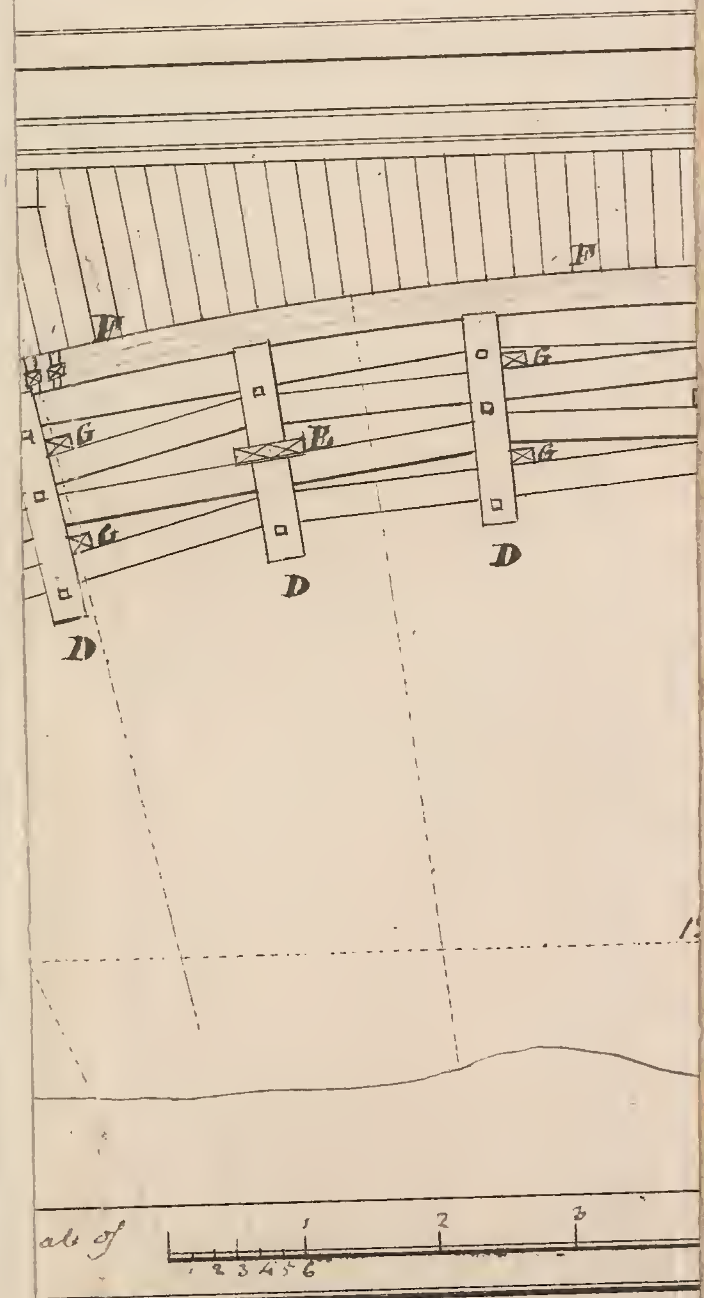
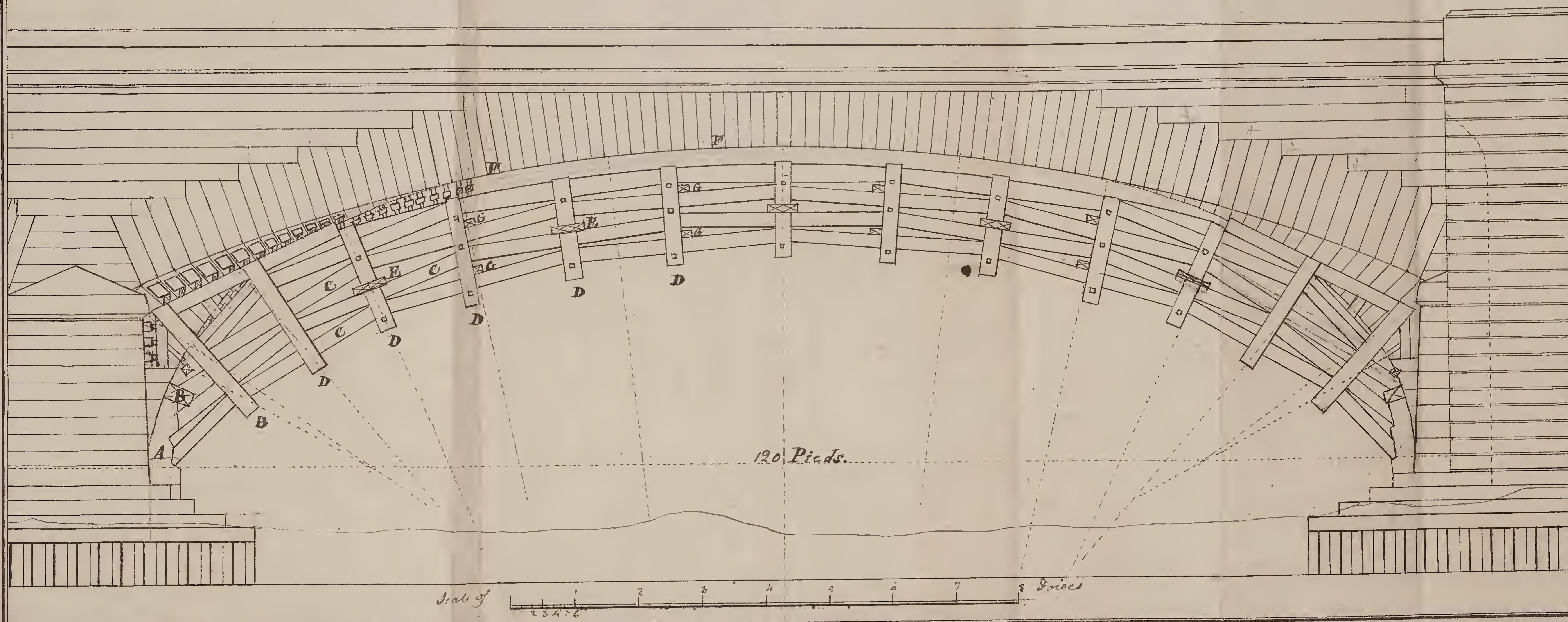


DESSEIN D'UNE DES CINQ ARCHES DU PONT DE NEUILLY.



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